Gravity and Circles Packet - Ans

1. (II) A box of mass 5.0 kg is accelerated by a force across a floor at a rate of $2.0\,\text{m/s}^2$ for 7.0 s. Find the net work done on the box.

Since the acceleration of the box is constant, use Eq. 2-11b to find the distance moved. Assume that the box starts from rest.

$\Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(2.0\,\text{m/s}^2\right)\left(7\text{ s}\right)^2 = 49\text{ m}$

Then the work done in moving the crate is

$W = F\Delta x\cos 0^\circ = ma\Delta x = \left(5\text{ kg}\right)\left(2.0\,\text{m/s}^2\right)\left(49\text{ m}\right) = \boxed{4.9\times10^2\,\text{J}}$

2. (I) How much work must be done to stop a 1250-kg car traveling at $105\,\text{km/h}$?

The work done on the car is equal to the change in its kinetic energy, and so

$$W = KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}\left(1250\,\text{kg}\right)\left[\left(105\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = \boxed{-5.32\times10^5\,\text{J}}$$

3. (I) A spring has a spring stiffness constant, *k*, of $440\,\text{N/m}$. How much must this spring be stretched to store 25 J of potential energy?

The elastic PE is given by $PE_{\text{elastic}} = \frac{1}{2}kx^2$ where *x* is the distance of stretching or compressing of the spring from its natural length.

$$x = \sqrt{\frac{2PE_{\text{elastic}}}{k}} = \sqrt{\frac{2\left(25\text{ J}\right)}{440\,\text{N/m}}} = \boxed{0.34\text{ m}}$$

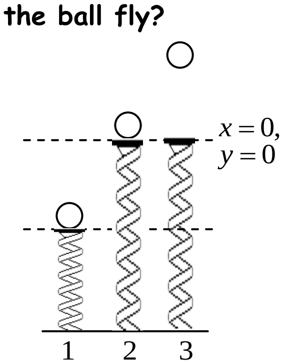
4. (I) By how much does the gravitational potential energy of a 64-kg pole vaulter change if his center of mass rises about 4.0 m during the jump?

Subtract the initial gravitational PE from the final gravitational PE.

$\Delta PE_{\text{grav}} = mgy_2 - mgy_1 = mg\left(y_2 - y_1\right) = \left(64\text{ kg}\right)\left(9.8\,\text{m/s}^2\right)\left(4.0\text{ m}\right) = \boxed{2.5\times10^3\,\text{J}}$

5. (II) A vertical spring (ignore its mass), whose spring stiffness constant is $950\,\text{N/m}$, is attached to a table and is compressed down 0.150 m. (*a*) What upward speed can it give to a 0.30-kg ball when released? (*b*) How high above its original position (spring compressed) will the ball fly?

Use conservation of energy. The level of the ball on the uncompressed spring taken as the zero location for both gravitational PE $\left(y=0\right)$ and elastic PE $\left(x=0\right)$. Take up to be positive for both.



(*a*) Subscript 1 represents the ball at the launch point, and subscript 2 represents the ball at the location where it just leaves the spring, at the uncompressed length. We have $v_1 = 0$, $x_1 = y_1 = -0.150\,\text{m}$, and $x_2 = y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0 + 0 \quad \rightarrow \quad v_2 = \sqrt{\frac{kx_1^2 + 2mgy_1}{m}}$$

$$v_2 = \sqrt{\frac{(950\,\text{N/m})(0.150\text{ m})^2 + 2(0.30\text{ kg})(9.80\,\text{m/s}^2)(-0.150\text{ m})}{(0.30\text{ kg})}} = \boxed{8.3\,\text{m/s}}$$

(*b*) Subscript 3 represents the ball at its highest point. We have $v_1 = 0$, $x_1 = y_1 = -0.150\,\text{m}$, $v_3 = 0$, and $x_3 = 0$. Solve for y_3.

$$E_1 = E_3 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = 0 + mgy_2 + 0 \quad \rightarrow \quad y_2 - y_1 = \frac{kx_1^2}{2mg} = \frac{(950\,\text{N/m})(0.150\text{ m})^2}{2(0.30\text{ kg})(9.80\,\text{m/s}^2)} = \boxed{3.6\text{ m}}$$

6. (II) A shot-putter accelerates a 7.3-kg shot from rest to $14\,\text{m/s}$. If this motion takes 1.5 s, what average power was developed?

The work done in accelerating the shot put is given by its change in kinetic energy: The power is the energy change per unit time.

$$P = \frac{W}{t} = \frac{KE_2 - KE_1}{t} = \frac{\tfrac{1}{2}m\left(v_2^2 - v_1^2\right)}{t} = \frac{\tfrac{1}{2}(7.3\text{ kg})\left[(14\,\text{m/s})^2 - 0\right]}{1.5\text{ s}} = 476.9\text{ W} \approx \boxed{4.8\times10^2\,\text{W}}$$