

**Unit 6: Work and Energy**

Wikipedia says (correctly) "In physics, **work** is the energy transferred to or from an object via the **application of force along a displacement.**" Work and energy are essentially interchangeable. Either can be converted to the other, and they both have the same units, Joules (J).

Energy is often defined as "the ability to do work." Two basic types of energy are kinetic energy (energy something has because it is in motion), and potential energy (stored energy). Both types of energy can be used to do work, and both types of energy can be *produced* by doing work.

Give some examples of energy being converted to work and work being converted to energy.

Work converted to kinetic energy:

Someone pushes  $\swarrow F$  a sled horizontally over a distance  $\searrow d$ .  
 moves (has KE)

Kinetic energy converted to work:

A moving  $\swarrow KE$  car pushes  $\swarrow F$  over a fire hydrant, and slows down in the process.

Work converted to potential energy:

Someone moves  $\swarrow F$  a box from the floor to a table top.  $\leftarrow$  height gained

Potential energy converted to work:

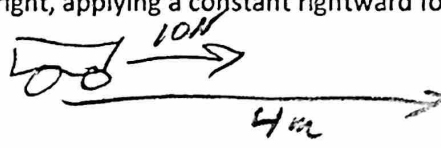
A ball is dropped.  $\nwarrow F_{gravity}$

Work can be calculated using the formula  $W=Fd$ . In the formula,  $d$  is the displacement (or distance) over which the force acts, and  $F$  is a force (or component of a force) in the direction of movement

Work Practice:

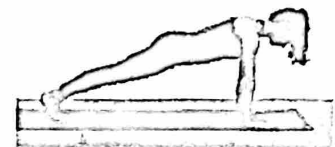
1. A child pulls a wagon 4m to the right, applying a constant rightward force of 10N. How much work does the child do?

$W = Fd$   
 $W = 10N(4m) = 40J$

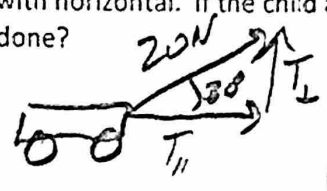


- 1.5. A 60kg military cadet holds a plank for 10 seconds. How much work does she do? [Follow the strict physics definition of work]

$W = Fd$   
 $W = 588N(0m) = 0J$   
 None -- no distance



2. Another child pulls a wagon using a rope. The tension in the rope is 20N, and the rope makes a 30° angle with horizontal. If the child applies this force constantly as the wagon travels 6m, how much work is done?



$$T_{\parallel} = 20N (\cos 30^{\circ}) = 17.3N$$

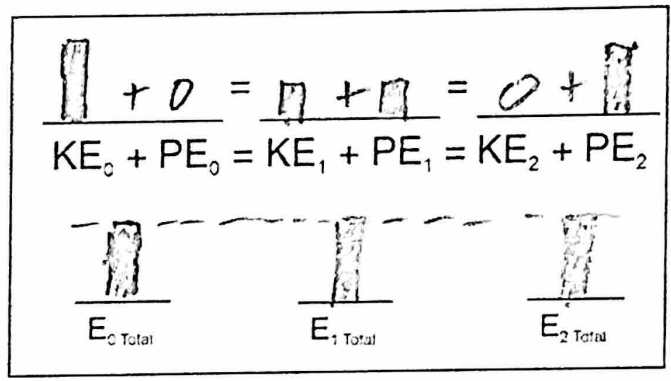
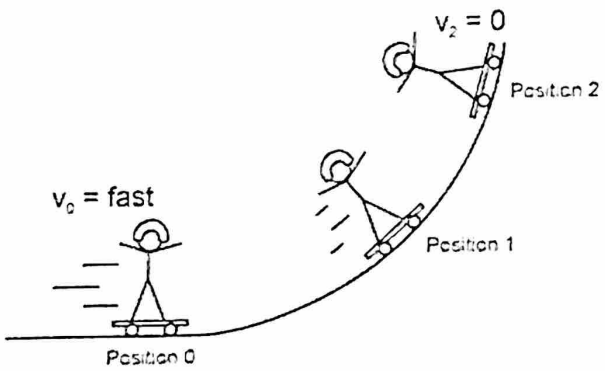
$$W = F_{\parallel} d = 17.3N (6m) = 104J$$

**Mechanical Energy:** energy determined by an object's motion or position. Examples that we will work with during this unit are kinetic energy, gravitational potential energy, and spring potential energy.

**Thermal Energy:** energy relating to an object's temperature, which is determined by the speed of its randomly-moving individual molecules. Heat is the flow (or transfer) of thermal energy from one object to another.

**Law of Conservation of Energy (\*\*for mechanical energy only\*\*):** Unless *mechanical energy* is being added to or removed from a system by work, the total amount of *mechanical energy* in a closed system remains constant. A simple equation expressing this is  $KE_0 + PE_0 = KE + PE$  (or  $KE_{initial} + PE_{initial} = KE_{final} + PE_{final}$ ). The total mechanical energy remains constant, so energy is said to be "conserved." In this situation, "conserved" means "total remains constant." This is a simple form of the **Law of Conservation of Energy**.

Use vertical bars to show how the relative values of the skateboarder's KE and PE vary at positions 0, 1, and 2.



**Law of Conservation of Energy (for all energy):** For any closed system,  $KE_i + PE_i + OE_i = KE_f + PE_f + OE_f$ . OE represents "other energy." Other energy can be chemical, electrostatic, thermal...

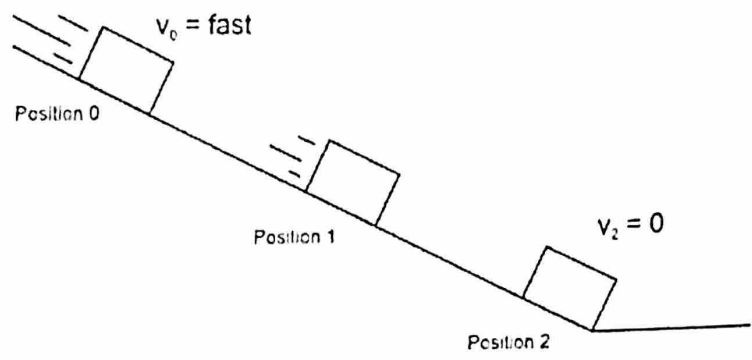
**Adding to – or Subtracting From – a system's total Mechanical Energy:** Often work is done to add or remove mechanical energy. This work is said to be done by "non-conservative forces," because the total amount of mechanical energy in the system changes – total mechanical energy is *not conserved*. **Work by non-conservative forces** is labeled  $W_{nc}$ . A more general equation for mechanical energy takes this work into account...

$$KE_{initial} + PE_{initial} + W_{nc} = KE_{final} + PE_{final}$$

When friction slows something down,  $W_{nc}$  is a negative number, because friction opposes motion. When something adds energy, its work ( $W_{nc}$ ) is a positive number. [Note that, in the case of friction, the energy is not really lost, but rather it gets turned into thermal energy. The equation above applies to mechanical energy, not thermal energy.]

**Example -- Negative Work by a Non-conservative Force:** A box is sliding down a ramp, slowing down at a constant rate until it stops.

- In the top space, use vertical bars to show the relationship between KE, PE, and non-conservative work.
- Identify the source of the non-conservative work.
- In the bottom spaces, use vertical bars to represent the relative values of the system's KE, PE, and OE at various stages in its slide.
- Identify the form of OE in this scenario.



### Changes in Mechanical Energy

$\text{KE}_0 + \text{PE}_0 + W_{\text{NC}} = \text{KE}_1 + \text{PE}_1$

$\text{Total Mechanical } E_0 \quad \text{Total Mechanical } E_1$

*Work removes energy*

$\text{KE}_1 + \text{PE}_1 + W_{\text{NC}} = \text{KE}_2 + \text{PE}_2$

$\text{Total Mechanical } E_1 \quad \text{Total Mechanical } E_2$

*Work removes energy*

*Friktion does negative work, decreasing total mechanical energy*

### Conservation With All Forms of Energy

Heat

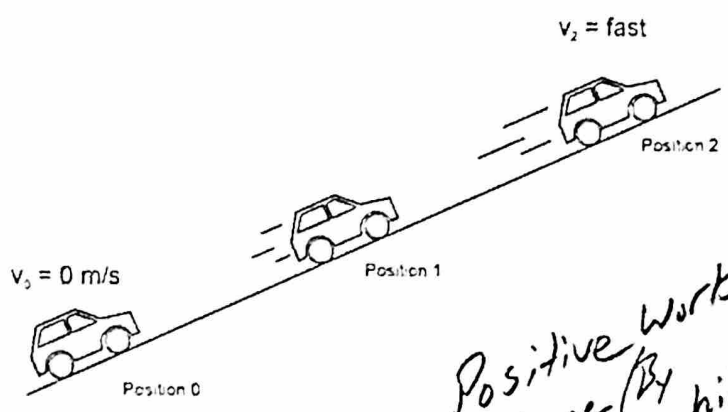
$\text{KE}_0 + \text{PE}_0 + \text{OE}_0 = \text{KE}_1 + \text{PE}_1 + \text{OE}_1 = \text{KE}_2 + \text{PE}_2 + \text{OE}_2$

$\text{Total } E_0 \quad \text{Total } E_1 \quad \text{Total } E_2$

*Total E is always conserved (stays the same) in a closed system.*

**Example -- Positive Work by a Non-conservative Force:** Starting from rest, a car accelerates up a hill at a constant rate.

- In the top space, use vertical bars to show the relationship between KE, PE, and non-conservative work.
- Identify the source of the non-conservative work.
- In the bottom spaces, use vertical bars to represent the relative values of the system's KE, PE, and OE at various stages in its slide.
- Identify the form of OE in this scenario.



*Positive work done (by engine, hills push, thrust...)*

### Changes in Mechanical Energy

$$0 + 0 + \text{Work} = \text{KE}_1 + \text{PE}_1$$


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$$KE_0 + PE_0 + W_{NC} = KE_1 + PE_1$$

Work adds energy

$$\text{KE}_1 + \text{PE}_1 + \text{Work} = \text{KE}_2 + \text{PE}_2$$


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$$KE_1 + PE_1 + W_{NC} = KE_2 + PE_2$$

Work adds energy

### Conservation With All Forms of Energy

$$0 + 0 + \text{Chemical or Electric Energy} = \text{KE}_1 + \text{PE}_1 + \text{OE}_1 = \text{KE}_2 + \text{PE}_2 + \text{OE}_2$$


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$$KE_0 + PE_0 + OE_0 = KE_1 + PE_1 + OE_1 = KE_2 + PE_2 + OE_2$$

Total E<sub>0</sub>

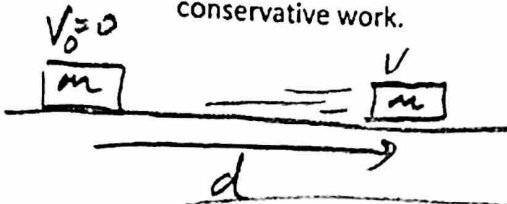
Total E<sub>1</sub>

Total E<sub>2</sub>

Deriving the Other Kinetic Energy formula:

Consider an object of mass  $m$  being accelerated from rest over a horizontal displacement  $d$ . It could be anything - a car, a block of wood, a baby lobster...

- First, derive what is known as the "work-energy theorem" by using the equation involving non-conservative work.



$$PE_0 + KE_0 + W_{NC} = PE + KE$$

$$0 + 0 + W_{NC} = 0 + KE$$

$W_{NC} = KE$  → more generally,  $W = \Delta KE$

- Second, derive an equation for the KE of this object in terms of its mass  $m$  and velocity  $v$ .

$$W = Fd \Rightarrow Fd = KE \Rightarrow mad = KE \Rightarrow KE = ma\left(\frac{1}{2}at^2\right)$$

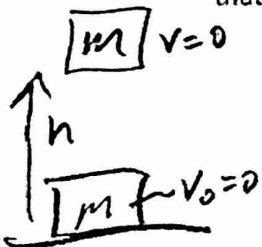
$$W = KE \Rightarrow F = ma \Rightarrow d = \Delta x = v_0 t + \frac{1}{2}at^2$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v}{t} \leftarrow v_0 = 0$$

$$KE = \frac{1}{2}mv^2 \leftarrow KE = \frac{1}{2}mt^2\left(\frac{v}{t}\right)^2 \leftarrow KE = \frac{1}{2}mt^2a^2$$

The Work-Energy Theorem can be useful, but it can also be tricky to apply. If you want to use it, it is technically  $W_{net} = \Delta KE$ . The net amount of work done on an object equals the object's change in KE. [Here's an example of its trickiness... if you lift a box from the floor and set it on a table, its KE has not changed, so there is no net work done on the box. At first this seems wrong, but it's actually right; you do positive work on the box and gravity does the same amount of negative work on the box. The total (net) work is zero.]

Deriving the Gravitational Potential Energy formula: Find the potential energy stored in an object of mass  $m$  that due to its having been lifted a height  $h$ .



$$PE_0 + KE_0 + W_{NC} = PE + KE$$

More generally,  $\Delta PE = mgh$

$$0 + 0 + W_{NC} = PE + 0$$

$$W = Fd = (mg)h$$

$mgh = PE$

Power is the rate of work.  $P = \frac{W}{t}$ . The units for power that we will use are Watts. 1 Watt = 1J/s.  
1horsepower = 746W