## Chapter Summary

### 9.2. The Second Condition for Equilibrium

- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.
10.1. Angular Acceleration
- Describe uniform circular motion.
- Explain non-uniform circular motion.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.
10.2. Kinematics of Rotational Motion
- Observe the kinematics of rotational motion.
- Derive rotational kinematic equations.
- Evaluate problem solving strategies for rotational kinematics.
10.3. Dynamics of Rotational Motion: Rotational Inertia
- Understand the relationship between force, mass and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.
10.4. Rotational Kinetic Energy: Work and Energy Revisited
- Derive the equation for rotational work.
- Calculate rotational kinetic energy.
- Demonstrate the Law of Conservation of Energy.
10.5. Angular Momentum and Its Conservation
- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.


## Key Equations

$s=\theta r \quad v=\omega r a_{t}=\alpha r a_{r}=a_{c}=\frac{v^{2}}{r}=\omega^{2} r$
$\omega_{\text {avg }}=\frac{\theta_{f}-\theta_{0}}{t} \quad \alpha_{\text {avg }}=\frac{\omega_{f}-\omega_{0}}{t}$
$\theta=\theta_{i}+\omega_{i} \dagger+\frac{1}{2} \alpha \dagger^{2} \quad \omega^{2}=\omega_{i}^{2}+2 \alpha\left(\theta_{f}-\theta_{i}\right) \quad \omega=\omega_{i}+\alpha \dagger$
$K_{\text {rot }}=\frac{1}{2} I \omega^{2}$
$I=\sum_{i=1}^{n} m_{i} r_{i}^{2}$ (discrete masses)
$\tau=r F=I \alpha \quad L=r p=r m v=I \omega \quad L_{i}=L_{f} \quad I_{i} \omega_{i}=I_{f} \omega_{f}$
For equilibrium: $\tau_{c w}=\tau_{c c w}$ and $\vec{F}_{\text {net }}=0$ (i.e $F_{x n e t}=0$ and $F_{y}=0$ )

## Multiple Choice

- Balancing beams
- Relationship of $\tau$ to $r$, F, I and $\alpha$
- Units for all the parameters
- Ways to increase (or decrease) torque
- Use of kinematic equations to determine $s, t, \omega, \alpha, \theta$
- Relationship of linear parameters $s, v, a_{t}, a_{r}$ to rotational parameters $\theta, \omega, \alpha$
- Which shape wins the race down a ramp
- Determining angular momentum
- Which configuration has more rotational inertia
- Definition of static equilibrium


## Problems

- Taken from practice problems in class


## Chapter 9-10 Practice Test

## Part I. Multiple Choice

1. With the same non-zero clockwise torque applied, if an object's rotational inertia is increased, its angular acceleration
A. increases.
B. decreases.
C. stays the same.
2. The torque applied to a bolt that is stuck can be increased by
A. increasing the length of the lever arm.
B. increasing the magnitude of the applied force.
C. changing the direction of the force to be perpendicular to the lever arm.
D. All of the above.
3. The units of angular speed are
A. $\mathrm{kg} \mathrm{m}^{2}$
B. rad
C. $\mathrm{rad} / \mathrm{s}$
D. $\mathrm{rad} / \mathrm{s}^{2}$
E. $s^{-1}$
4. The units of angular acceleration are
A. $\mathrm{kg} \mathrm{m}^{2}$
B. rad
C. $\mathrm{rad} / \mathrm{s}$
D. $\mathrm{rad} / \mathrm{s}^{2}$
E. $s^{-1}$
5. A wheel with a radius of 0.500 m rotates through an angle of $4 \pi$. How far has a point on the outer rim traveled?
A. 0.500 m
B. 3.14 m
C. 6.28 m
D. 9.42 m
E. 25.1 m
6. In an effort to tighten a bolt, a force $F$ is applied as shown in the figure below. If the distance from the end of the wrench to the center of the bolt is 25.0 cm and $F=5.00 \mathrm{~N}$, what is the magnitude of the torque produced by $F$ ?
A. 0.00 Nm
B. 1.25 Nm
C. 5.00 Nm
D. 75.0 Nm
E. 125.0 Nm

7. If a wheel turning at a constant rate completes exactly 100 revolutions in 10.0 s , its angular speed is:
A. $0.314 \mathrm{rad} / \mathrm{s}$
B. $0.628 \mathrm{rad} / \mathrm{s}$
C. $10.0 \mathrm{rad} / \mathrm{s}$
D. $62.8 \mathrm{rad} / \mathrm{s}$
E. $314 \mathrm{rad} / \mathrm{s}$
8. A child initially standing on the edge of a freely spinning merry-go-round moves to the center. Which one of the following statements is necessarily true concerning this event and why?
A. The angular speed of the system increases because the moment of inertia of the system has decreased.
B. The angular speed of the system decreases because the moment of inertia of the system has decreased.
C. The angular speed of the system increases because the moment of inertia of the system has increased.
D. The angular speed of the system decreases because the moment of inertia of the system has increased.
$E$. The angular speed of the system remains the same because the net torque on the merry-go-round is zero.
9. When a spinning ice skater draws in her outstretched arms, her kinetic energy increases. Where does this added energy come from?
A. The added energy comes from a decrease in her angular momentum.
B. The added energy comes from the dark energy of the vacuum.
C. The added energy comes from the work the skater does pulling in her arms.
D. The added energy comes from the decreased angular speed.
$E$. The added energy is transferred from the ice to the skater.
10. In order to increase the torque created by $F_{1}$ below, the fulcrum should be moved
A. closer to $F_{1}$
B. closer to $\mathrm{F}_{2}$

11. Which one of the following statements provides the best definition of rotational inertia?
A. Rotational inertia is the momentum of a rotating object.
B. Rotational inertia is equal to the mass of the rotating object.
C. Rotational inertia is the resistance of an object to a change in its angular velocity.
D. Rotational inertia is the resistance of an object to a change in its linear velocity.
12. For a body to be in equilibrium, what conditions must apply?
A. $\tau_{c w}=0$ and $\tau_{c c w}=0$
B. $F_{x \text { net }}=F_{y \text { net }}$
C. $\tau_{c w}=\tau_{c c w}$ and $F_{x \text { net }}=F_{y \text { net }}$
D. $\tau_{c w}=\tau_{c c w}, F_{x \text { net }}=0$ and $F_{y \text { net }}=0$
13. A comet orbiting the Sun can be considered an isolated system with no outside forces or torques acting on it. As the comet moves in its highly elliptical orbit, what remains constant?
A. Its distant from the Sun
B. Its angular speed
C. Its linear speed
D. Its angular momentum
E. The gravitational force between the comet and the Sun.
14. A uniform disk, a hoop and a uniform solid sphere are held stationary at the top of a ramp. All 3 objects have the same mass and radius. When released, they roll down the ramp without slipping. Rank the objects according to their speed at the bottom of the incline from least to greatest.

$$
I_{\text {disk }}=1 / 2 M R^{2} \quad I_{\text {hoop }}=M R^{2} \quad I_{\text {solid sphere }}=2 / 5 M R^{2}
$$

A. hoop, disk, sphere
C. sphere, hoop, disk
B. disk, hoop, sphere
D. sphere, disk, hoop
15. A disk initially rolls along the flat ground at a constant speed without slipping. If linear speed of the disk is now doubled,
A. the angular speed is increases by $2 X$ and the kinetic energy increases by $2 X$.
B. the angular speed is increases by $4 X$ and the kinetic energy increases by $2 X$.
C. the angular speed is increases by $2 X$ and the kinetic energy increases by $4 X$.
D. the angular speed is increases by $4 X$ and the kinetic energy increases by $4 X$.
$E$. both the angular speed and the kinetic energy remain the same.
16. What happens when a spinning ice skater draws in her outstretched arms?
A. Her moment of inertia decreases causing her to speed up.
B. Her angular momentum decreases.
C. The torque that she exerts increases her moment of inertia.
D. Her angular momentum increases.
E. Her moment of inertia decreases causing her to slow down.
17. A 3 kilogram mass is attached 2 meters from a rotating rod. What is the rotational inertia of the mass?
A. $12 \mathrm{~kg} \mathrm{~m}^{2}$
B. $54 \mathrm{kgm}^{2}$
C. $6 \mathrm{kgm}^{2}$
D. $18 \mathrm{~kg}^{2}$
E. $36 \mathrm{~kg}^{2}$
18. A torque applied to a solid object that is free to move will produce
A. a linear acceleration.
B. rotational equilibrium.
C. an angular acceleration.
D. rotational inertia.
19. Assume this bar has mass $M$ and forces $F_{1}$ and $F_{2}$ are pulling down on each end. In order for the bar to be balanced:
A. $F_{1}=F_{2}$
B. $F_{1}<F_{2}$
C. $r_{1} F_{1}=r_{2} F_{2}$
D. $\tau_{c w}=0$ and $\tau_{c c w}=0$

E. $r_{1} F_{1}=r_{c M} M g+r_{2} F_{2}$
II. Problems: On a separate sheet of paper, show your starting equation(s), show your work and box your answer.
(5 points each:

| Starting equation: | 1 point |
| :--- | :--- |
| Work and correct answer: | 3.5 points |
| Boxed answer: | 0.5 points) |

1. An ultracentrifuge accelerates from rest to a top speed of 1870 revolutions per second in 2.00 min .
A. What is its angular acceleration in $\mathrm{rad} / \mathrm{s}^{2}$ ?
B. What is the tangential acceleration at a point 7.00 cm from the axis of rotation?
C. What is the radial (i.e. centripetal) acceleration at a point 7.00 cm from the axis of rotation when the centrifuge is rotating at top speed?
2. A drum rotates around its central axis at an initial angular speed of $11.50 \mathrm{rad} / \mathrm{s}$. The drum then slows at a constant rate of $2.60 \mathrm{rad} / \mathrm{s}^{2}$.
A. How much time does it take to come to a stop?
B. Through what angle does it rotate before coming to a stop?
3. Suppose you exert a 1450 N force tangent to the surface of a solid sphere that has a radius of 0.340 meters and a mass of 13.0 kg . $I_{\text {solid sphere }}=2 / 5 \mathrm{MR}^{2}$.
A. What torque is exerted?
B. What is the angular acceleration assuming negligible opposing friction?
4. Ice Skater
A. Calculate the angular momentum of an ice skater spinning at $3.60 \mathrm{rev} / \mathrm{s}$ given that her moment of inertia is $0.320 \mathrm{~kg} \cdot \mathrm{~m}^{2}$.
B. She then reduces her rate of spin (her angular velocity) by extending her arms.

Find the new value of her moment of inertia if her angular velocity decreases to $1.20 \mathrm{rev} / \mathrm{s}$.
5. A 0.440-kg soccer ball with a radius of 11.2 cm is initially at rest at a height of 3.90 meters at the top of a ramp. It is then released. The soccer ball can be thought of as a hollow spherical shell, $I=\frac{2}{3} M R^{2}$.
A. What is the final velocity of the soccer ball at the bottom of the ramp if the ramp is frictionless, so the ball just slides down the ramp without rolling?
B. What is the final velocity of the soccer ball at the bottom of the ramp if there is friction and it rolls down the ramp without slipping?
6. What force needs to be applied at the end of a uniform 5.0-meter beam to keep it level. A 240 N mass is hung 2.0 meters from the fulcrum. The beam weighs 600 N .


