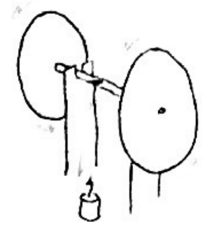
Physics 200 Name: Key

Rubber Band Car Practice Test (probably 4 problems longer than the actual test)



Part 1: **Wheel and Axle Testing** – Students tie one end of a string to their car's drive axle, and they tie the other end of the string to a weight. They wrap the weight around the axle, and then release the weight and axle from rest, while holding the car's frame steady. The data in the table correspond to the weight's first trip down and up. These data also correspond to the car's non-drive axle, since the students chose to make the front and rear wheels identical.

Wheel radius (m)	0.1
Mass of falling weight (kg)	0.2
Distance descended by weight (m)	0.63
Descent time (s)	3.2
Distance risen by weight before stopping (m)	0.51
Number of wheel and axle rotations during descent	24.5

1. Angular displacement during the weight's descent = 154 rad

$$24.5\,\cancel{\text{rotations}}\left(\frac{2\pi\,\text{rad}}{\cancel{\text{rot}}}\right) =$$

2. Axle radius = 0.00409 m

$$\boxed{s = \theta r} \quad 0.63\,\text{m} = 154\,\text{rad}\,(r)$$

3. Angular acceleration during the weight's descent = 30.1 rad/s²

$$\boxed{\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2} \quad 154\,\text{rad} = 0 + \tfrac{1}{2}\alpha(3.2\,\text{s})^2$$

4. Linear acceleration of the falling mass during its descent = 0.123 m/s²

$$\boxed{a = \alpha r} = 30.1\,\text{rad/s}^2(0.00409\,\text{m}) = 0.123\,\text{m/s}^2$$

5. String tension during the weight's descent = 1.94N

$$\boxed{T = m(a+g)} \quad T = 0.2\,\text{kg}(-0.123\,\text{m/s}^2 + 9.8\,\text{m/s}^2)$$

6. Angular displacement during the entire round trip = 279 rad

$$\boxed{s = \theta r} \quad 0.63\,\text{m} + 0.51\,\text{m} = \theta(0.00409\,\text{m})$$

7. Total work done by friction during the entire round trip = −0.2352J

$$PE_0 + KE_0 + W_{NC} = PE + KE$$

$$0.2\,\text{kg}(9.8\,\text{m/s}^2)(0.63\,\text{m}) + 0 + W_{NC} = 0.2\,\text{kg}(9.8\,\text{m/s}^2)(0.51\,\text{m}) + 0$$

$$W_{NC} = -0.2352\,\text{J}$$

8. Average torque due to friction during the entire round trip = $-0.00844\ Nm$

$\boxed{W = \tau \Delta\theta}$ $\quad -0.2352\,J = \tau_{Fr}(279\,rad)$

9. Torque applied by string tension during the descent = $0.00792\ Nm$

$\boxed{\tau = Fr}$ $= 1.94N(0.00409m)$

tension

10. Net torque during the descent = $0.00708\ Nm$

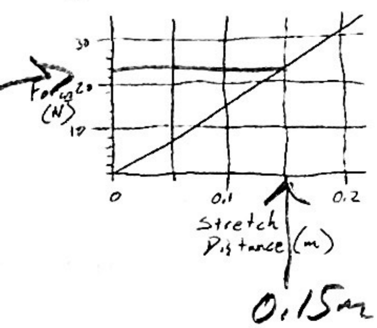
$\tau_{Net} = \tau_{String} + \tau_{Fr} = 0.00792Nm - 0.00844Nm$

11. Moment of inertia of the wheel and axle = $0.000235\ kg\,m^2$

$\boxed{\tau_{Net} = I\alpha}$ $\quad 0.00708Nm = I(30.1\ rad/s^2)$

Part 2: **Motor Design and Testing** -- The students design a motor that they think will give the car the most energy. They conduct some tests to find out how far they can wind the motor before the wheels slip (and bad things happen). To do this, they do some "false starts," letting the car start to move, but catching it immediately after it starts to move. They find that 0.15m is as far as they can stretch the bands without exceeding the wheels' maximum static friction with the floor. This also means that 0.15m of string unwinds from the axle while the bands are pulling (and while the car is accelerating).

After they determine the rubber band stretch distance, they measure the motor's energy input...



12. The force curve on the right represents the force required to stretch their rubber band motor to different lengths. What is the maximum force that they apply when they stretch their motor 0.15m?

Max Stretching Force = 24 N

13. What is the average force they apply to the rubber band motor as they stretch it to this distance?

Average Stretching Force = 12N $\quad F_{Ave} = \dfrac{F_0 + F}{2} = \dfrac{0 + 24}{2}$

14. They put energy into the motor by stretching the bands and doing work. What is the input energy that they give to the motor?

Motor Input Energy = 1.8 J

Input Work $\Rightarrow W = F_{Ave}\, d = 12N(0.15m) = 1.8J$

Then they measure the motor's output...

15. The students wind up the car so that the rubber bands are stretched the full 0.15m. They hold the drive wheels off the ground and let the motor freely accelerate the drive wheels. Slow motion video (240 frames per second) of this event shows that, when the wheels reach their maximum speed, they make 1 full rotation for every 15 frames.

video frames.

a. The wheels' maximum rotational speed = 100.5 rad/s

$$\omega_{max} = \frac{1 \text{ rotation}}{15 \text{ frames}}\left(\frac{2\pi \text{ rad}}{1 \text{ rot}}\right)\left(\frac{240 \text{ frames}}{1 \text{ sec}}\right) = 100.5 \text{ rad/s}$$

b. Motor Output Energy = 1.19 J

$$\boxed{KE_{rot} = \tfrac{1}{2} I \omega^2} = \tfrac{1}{2}(0.000235 \text{ kg m}^2)(100.5 \text{ rad/s})^2$$

16. Motor % efficiency = 66.1%

$$\% \text{ Efficiency} = \frac{\text{Output } E}{\text{Input } E} \times 100\% = \frac{1.19 J}{1.8 J} \times 100\%$$

Part 3: **Acceleration Distance Prediction**

17. Drive wheel angular displacement during acceleration = 36.7 rad

$$\boxed{S = \Theta r} \quad 0.15 m = \Theta (0.00409 m)$$

band stretch = string length

axle

18. Distance car will roll during acceleration = 3.67 m

$$\boxed{S = \Theta r} \quad S = 36.7 \text{ rad}(0.1 m)$$

wheel radius

Part 4: **Speed Prediction** -- Assuming that the entire car will receive the same actual output energy that you just calculated, find the speed that the car will reach at the end of its acceleration. You'll need the extra information in the table.

Car total Mass (kg)	0.35
Front Wheel and Axle Moment of Inertia (kgm^2)	0.00005
Front wheel and axle radius (m)	0.05

f = front
D = Drive

19. Use words and symbols (no numbers at this point) to write a basic equation setting the motor output energy equal to all the individual "energies" that the car will have when it reaches its top speed.

$$\text{Output Energy} = \frac{1}{2} m_{total} v^2 + \frac{1}{2} I_f \omega_f^2 + \frac{1}{2} I_D \omega_D^2$$

20. Starting with the equation that you just wrote, find a way to replace angular velocity (ω) with an equivalent linear velocity (in terms of v). Solve algebraically for v, in terms of output energy, car mass, wheel and axle moment of inertia, and wheel radius.

$$E_1 = \frac{1}{2} m v^2 + \frac{1}{2} I_f\left(\frac{v^2}{r_f^2}\right) + \frac{1}{2} I_D\left(\frac{v^2}{r_D^2}\right)$$

$$2E = v^2\left(m + \frac{I_f}{r_f^2} + \frac{I_D}{r_D^2}\right) \Rightarrow v = \sqrt{\frac{2(\text{Output } E)}{m + \frac{I_f}{r_f^2} + \frac{I_D}{r_D^2}}}$$

21. Use your formula to calculate the car's top speed.

Top speed = 2.45 m/s

$$v = \sqrt{\frac{2(1.19\,J)}{0.35\,kg + \frac{0.00005\,kgm^2}{(0.05\,m)^2} + \frac{0.000235\,kgm^2}{(0.1\,m)^2}}}$$

Part 5: **And Now For Something Completely Different! – Spinning Figure Skaters and Balancing Torques**

21. A spinning figure skater has a moment of inertia of $3.04\,kgm^2$, and she is currently spinning at a rate of 200rpm. If she wants to speed up to 250rpm...

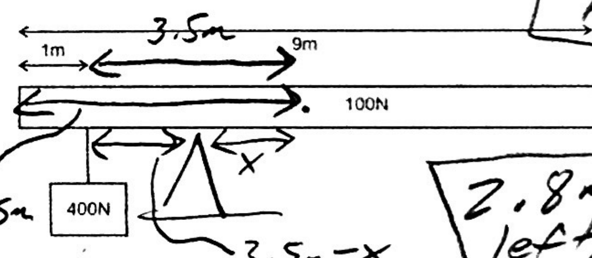
a. Describe what she needs to do with her body to make this happen.

Pull in her arms

b. What new moment of inertia will she have when she reaches 250rpm?

$$\boxed{L_i = L_f \quad I_i \omega_i = I_f \omega_f} \quad (3.04\,kgm^2)(200\,rpm) = I_f(250\,rpm) \Rightarrow \boxed{I_f = 2.43\,kgm^2}$$

22. A 9m long beam of uniformly distributed mass has a weight of 100N. There is an additional weight of 400N hanging from the beam at a point 1m from the left end of the beam. **Provide a precise description the location at which a fulcrum could be placed under the beam to cause the beam to balance horizontally.**



$$\tau_{ccw} = \tau_{cw}$$
$$400N(3.5m - x) = 100N(x)$$
$$1400Nm - 400Nx = 100Nx \Rightarrow 1400Nm = 500Nx$$
$$\boxed{x = 2.8m}$$

2.8m left of center