Circular Motion Problems #2
Answers to Vertical Motion Problems

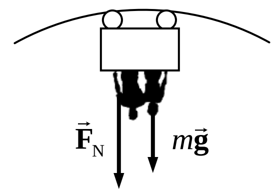
Problems

1. (II) At what minimum speed must a roller coaster be traveling when upside down at the top of a circle so that the passengers will not fall out? Assume a radius of curvature of 7.4 m.

At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's 2nd law for the passengers.

$$\sum F = F_N + mg = ma = mv^2/r \quad \rightarrow \quad F_N = m\left(v^2/r - g\right)$$

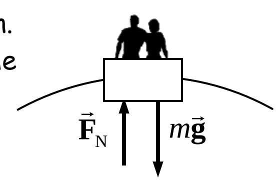
We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car – they are in free fall. The limiting condition is



$$v_{min}^2/r - g = 0 \quad \rightarrow \quad v_{min} = \sqrt{rg} = \sqrt{(9.8\,\text{m/s}^2)(7.4\text{ m})} = \boxed{8.5\,\text{m/s}}$$

2. (II) How many revolutions per minute would a 15-m-diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point?

The free-body diagram for passengers at the top of a Ferris wheel is as shown. F_N is the normal force of the seat pushing up on the passenger. The sum of the forces on the passenger is producing the centripetal motion, and so must be a centripetal force. Call the downward direction positive. Newton's 2nd law for the passenger is:



$$\sum F_R = mg - F_N = ma = mv^2/r$$

Since the passenger is to feel "weightless", they must lose contact with their seat, and so the normal force will be 0.

$$mg = mv^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{(9.8\,\text{m/s}^2)(7.5\,\text{m})} = 8.6\,\text{m/s}$$

$$\left(8.6\frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi(7.5\text{ m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{11\text{ rpm}}$$

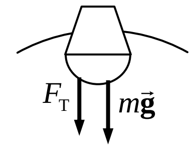
3. (II) A bucket of mass 2.00 kg is whirled in a vertical circle of radius 1.10 m. At the lowest point of its motion the tension in the rope supporting the bucket is 25.0 N. (*a*) Find the speed of the bucket. (*b*) How fast must the bucket move at the top of the circle so that the rope does not go slack?

At the bottom of the motion, a free-body diagram of the bucket would be as shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's 2nd law for the bucket, with up as the positive direction.

$$\sum F_R = F_T - mg = ma = mv^2/r \quad \rightarrow$$

$$v = \sqrt{\frac{r(F_T - mg)}{m}} = \sqrt{\frac{(1.10\text{ m})\left[25.0\text{ N} - (2.00\text{ kg})(9.80\,\text{m/s}^2)\right]}{2.00\text{ kg}}} = 1.723 \approx \boxed{1.7\,\text{m/s}}$$

(*b*) A free-body diagram of the bucket at the top of the motion is shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's 2nd law for the bucket, with down as the positive direction.



$$\sum F_R = F_T + mg = ma = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{r(F_T + mg)}{m}}$$

If the tension is to be zero, then

$$v = \sqrt{\frac{r(0+mg)}{m}} = \sqrt{rg} = \sqrt{(1.10\text{ m})(9.80\,\text{m/s}^2)} = \boxed{3.28\,\text{m/s}}$$

The bucket must move faster than 3.28 m/s in order for the rope not to go slack.

4. (II) How fast (in rpm) must a centrifuge rotate if a particle 9.00 cm from the axis of rotation is to experience an acceleration of 115,000 *g*'s?

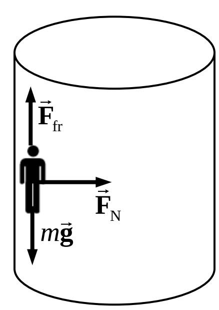
The centripetal acceleration of a rotating object is given by $a_R = v^2/r$. Thus

$$v = \sqrt{a_R r} = \sqrt{(1.15\times10^5 g)r} = \sqrt{(1.15\times10^5)(9.80\,\text{m/s}^2)(9.00\times10^{-2}\text{m})} = 3.18\times10^2\text{ m/s}.$$

$$(3.18\times10^2\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(9.00\times10^{-2}\text{m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{3.38\times10^4\,\text{rpm}}.$$

5. (II) In a "Rotor-ride" at a carnival, people are rotated in a cylindrically walled "room." (See Fig. 5–35.) The room radius is 4.6 m, and the rotation frequency is 0.50 revolutions per second when the floor drops out. What is the minimum coefficient of static friction so that the people will not slip down? People on this ride say they were "pressed against the wall." Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides "scary")? [*Hint*: First draw the free-body diagram for a person.]

Consider the free-body diagram for a person in the "Rotor-ride". $\vec{\mathbf{F}}_N$ is the normal force of contact between the rider and the wall, and $\vec{\mathbf{F}}_{fr}$ is the static frictional force between the back of the rider and the wall. Write Newton's 2nd law for the vertical forces, noting that there is no vertical acceleration.



$$\sum F_y = F_{fr} - mg = 0 \quad \rightarrow \quad F_{fr} = mg$$

If we assume that the static friction force is a maximum, then

$$F_{fr} = \mu_s F_N = mg \quad \rightarrow \quad F_N = mg/\mu_s.$$

But the normal force must be the force causing the centripetal motion – it is the only force pointing to the center of rotation. Thus $F_R = F_N = mv^2/r$. Using $v = 2\pi r/T$, we have

$F_N = \dfrac{4\pi^2 mr}{T^2}$. Equate the two expressions for the normal force and solve for the coefficient of friction.

Note that since there are 0.5 rev per sec, the period is 2.0 sec.

$$F_N = \frac{4\pi^2 mr}{T^2} = \frac{mg}{\mu_s} \rightarrow \mu_s = \frac{gT^2}{4\pi^2 r} = \frac{(9.8\,\text{m/s}^2)(2\text{ s})^2}{4\pi^2(4.6\text{ m})} = \boxed{0.22}.$$

Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, and so the period could be longer or the cylinder smaller.

There is no force pushing outward on the riders. Rather, the wall pushes against the riders, so by Newton's 3rd law the riders push against the wall. This gives the sensation of being pressed into the wall.

6. (III) A pilot performs an evasive maneuver by diving vertically at $310\,\text{m/s}$. If he can withstand an acceleration of 9.0 g's without blacking out, at what altitude must he begin to pull out of the dive to avoid crashing into the sea?

The fact that the pilot can withstand 9.0 *g*'s without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.

$$a_R = v^2/r = 9.0g \rightarrow r = \frac{v^2}{9.0g} = \frac{(310\,\text{m/s})^2}{9.0(9.80\,\text{m/s}^2)} = \boxed{1.1\times 10^3\,\text{m}}$$