Physics 200 (Stapleton)
Unit 5 Handout:
Circular Motion and The Law of Gravitation

Name: A5/6

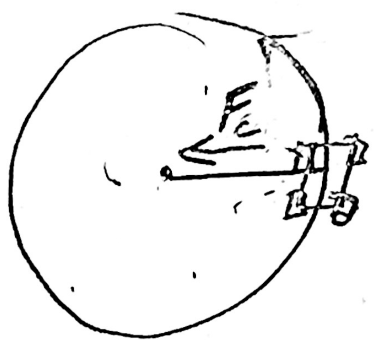
Circular Motion Formulas: *** These formulas only apply to objects undergoing "uniform circular motion" (i.e. **circular motion** at a **constant speed**

$a_{centripetal} = v^2/r$ $\quad\quad F_{net\ centripetal} = ma_{centripetal} = mv^2/r$

$a_{centripetal}$ is directed toward the center of the circle.

Circular Motion Example Problems:

1. (horizontal circle) A 500kg car drives in a circle with a radius of 20m. If the car maintains a constant speed of 20m/s, what centripetal force acts on the car? If the driving surface is flat and horizontal, what provides the centripetal force?



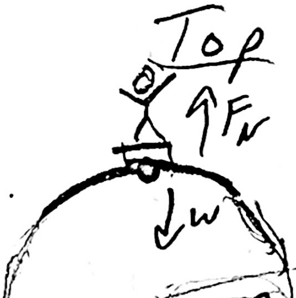
$$\Sigma F = \frac{mv^2}{r} = \frac{500kg\,(20 m/s)^2}{20m} = 10,000N$$

Toward the center

$$\Sigma F = F_c = F_{Friction}$$

2. (vertical circle) A 60kg teenager is riding a Ferris Wheel at the county fair. She is traveling in a uniform circular path with a radius of 4m, and her speed is constant at 7m/s. She is standing on a bathroom scale.
   a. What is the scale reading when she is at the top of the circle?
   b. What is the scale reading when she is at the bottom of the circle?

Top



$$\Sigma F = \frac{-mv^2}{r} = F_N - W$$

$$\frac{-60kg\,(7m/s)^2}{4m} = F_N - 588N$$

$$F_N = -147N$$

Oh no! F_N is negative, which means she flies off!

Bottom

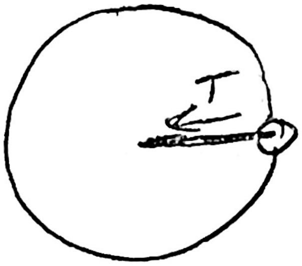
$$\Sigma F = \frac{mv^2}{r} = F_N - 588N$$

$$\frac{60kg\,(7m/s)^2}{4m} = F_N - 588N$$

$$F_N = 1323N$$

Circular Motion Practice Problems:

3. [Horizontal circles] A 0.4kg ball on a string is swinging in circles (in a horizontal plane) at a constant speed of 3m/s. The radius of the orbit (i.e. the string length) is 0.5m and the string is horizontal (because this is happening in the absence of gravity). What is the tension in the string?



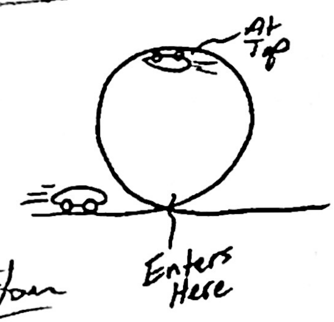
$\Sigma F = \frac{mv^2}{r} \Rightarrow T = \frac{mv^2}{r} = \frac{0.4kg\,(3m/s)^2}{0.5m}$

$\Sigma F = T$

$T = 7.2N$

4. [Vertical Circles] A car is approaching a "loop-the-loop" with a radius of 15m. What speed does the car need to maintain in order to experience a normal force at the top of the loop that is equal to the weight of the car? At this speed, what normal force does the car experience when it is just entering the loop?

1000kg



At Top

$\Sigma F \downarrow \quad \downarrow W \quad \downarrow F_N$

$\Sigma F = \frac{mv^2}{r} = -W - F_N$

Net force is negative, since it's toward the center, which is down.

$-\frac{mv^2}{r} = -W - W$

$-\frac{mv^2}{r} = -2mg$

$v = \sqrt{2gr}$

$v = \sqrt{2(9.8m/s^2)(15m)}$

$v = 17.2\,m/s$

At Bottom

$\Sigma F = \frac{mv^2}{r} = F_N - W$

$\frac{1000kg\,(17.2)^2}{15m} = F_N - 9800N$

$\Sigma F \uparrow \quad \uparrow F_N \quad \downarrow W$

$F_N = 29{,}400N$ (3x weight)

5. [Vertical Circles] A child weighing 200N is standing on a bathroom scale inside a Ferris Wheel that is rotating at a constant rate. If the radius of the circles made by the child is 10m, and the scale reads 100N at the top, what is the child's speed? What does the scale read when the child is at the bottom?

Top

$\uparrow F_N = 100N$

$m = 20.4kg$

$\Sigma F \downarrow \quad \downarrow W = 200N$

$\Sigma F = -\frac{mv^2}{r} = 100N - 200N$

Center of circle is downward

$-\frac{20.4kg\,(v^2)}{10m} = -100N$

$v = 7m/s$

Bottom

$\Sigma F \uparrow \quad \uparrow F_N$

$\downarrow W = 200N$

$\Sigma F = \frac{mv^2}{r} = F_N - 200N$

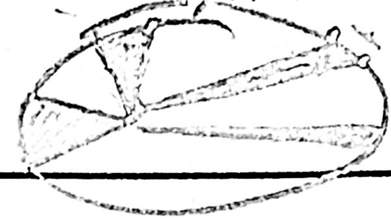
$\frac{20.4kg\,(7m/s)^2}{10m} = F_N - 200N$

$F_N = 300N$

Newton's Law of Universal Gravitation:

$F_{gravity} = G\left(\frac{m_1 m_2}{r^2}\right)$ --or-- $G\left(\frac{Mm}{r^2}\right)$, where G is the gravitational constant (an empirically measured quantity), m_1 and m_2 are two different masses, and r is the distance between their centers of mass. When one mass orbits the other, r is also referred to as the "orbital radius." *[Often, M is used for a planetary mass, and m is used for its satellite.]*

$G = 6.67x10^{-11}\frac{Nm^2}{kg^2}$



4. Calculate the force of gravity between a 100kg student and a 60kg student whose centers of mass are 1.7m apart.

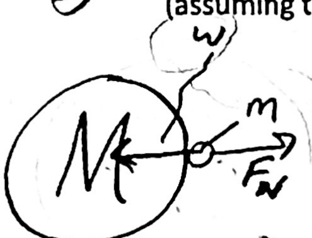
$$F_g = 6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}\left(\frac{(100kg)(60kg)}{(1.7m)^2}\right) = 1.38\times10^{-7}N$$

Combining Circular Motion and The Law of Gravitation:

5. Find the value of g at Earth's surface. Earth's mass is ($5.972x10^{24}$kg) and its average radius ($6.371x10^{6}$m).

$$g = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\left(\frac{5.972\times10^{24}kg}{(6.371\times10^{6}m)^2}\right) = 9.81m/s^2$$

6. Derive a general formula for the value of g at a distance r from the center of a planet with mass M (assuming that this location is at or above the planet's surface).



$$F_g = W = mg$$

$$F_g = G\frac{Mm}{r^2}$$

$$\cancel{m}g = G\frac{M\cancel{m}}{r^2}$$

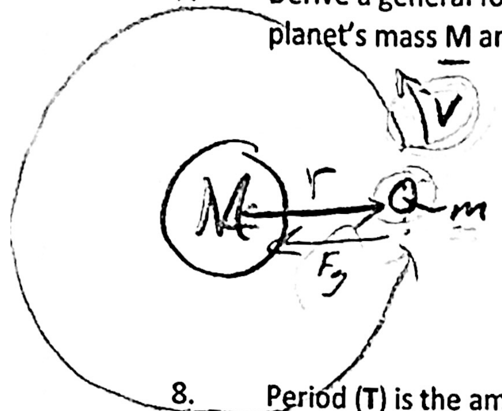
$$g = \frac{GM}{r^2}$$

8. What is the velocity of a space station that is orbiting ~~the Earth with~~ an orbital radius of 30,000km?

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}(5.972\times10^{24}kg)}{30{,}000{,}000m}}$$

$$V = 3644 m/s$$

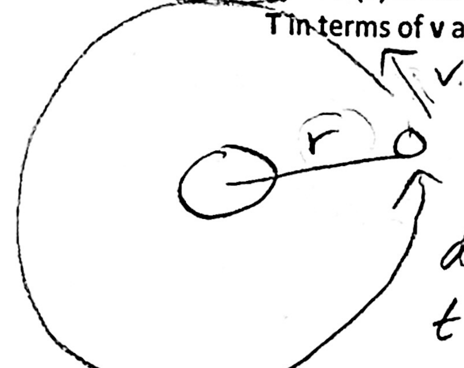
7. Derive a general formula for the speed **v** of a satellite in a circular orbit – in terms of the orbited planet's mass **M** and the satellite's orbital radius **r**.



$$\Sigma F = \frac{mv^2}{r} \qquad \frac{mv^2}{r} = G\frac{Mm}{r^2}$$

$$\Sigma F = F_g = G\frac{Mm}{r^2} \qquad v = \sqrt{\frac{GM}{r}}$$

8. Period (T) is the amount of time it takes for a satellite to complete a full orbit. Write equations for: T in terms of v and r; and v in terms of T and r.



$$T = \frac{2\pi r}{v} \qquad v = \frac{2\pi r}{T}$$

$$d = rt$$

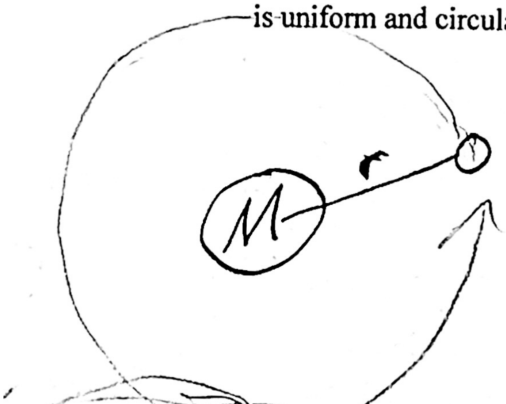
$$t = \frac{dist}{rate}$$

9. Find the necessary orbital radius for a geostationary satellite (a satellite that is always over the same point on the equator. You'll need the Earth's mass -- 5.972×10^{24}kg.

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow 86{,}400\,s = 2\pi\sqrt{\frac{r^3}{(6.67\times10^{-11}\,\frac{Nm^2}{kg^2})(5.972\times10^{24}\,kg)}}$$

$$r = 4.22\times10^7\,m$$

10. Derive a formula for **T** in terms of **r**, **G**, and the mass of the orbited body (**M**). Assume that the orbit is uniform and circular. *[This is the general form of Kepler's 3rd Law.]*



$$v = \sqrt{\frac{GM}{r}} \qquad \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$v = \frac{2\pi r}{T} \qquad T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}}$$

$$T^2 = \frac{4\pi^2 r^2}{\left(\frac{GM}{r}\right)}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \Leftarrow T^2 = \frac{4\pi^2 r^3}{GM} \Leftarrow$$

5 check#1

Physics 200 (Stapleton)

Big Quiz (Little Test): Circular Motion, Gravitation, Kepler

Name: Key

units -1/4

Conceptual Questions 13pts EABCCC DE FG

1. The gravitational force between two masses separated by a distance r is 400 N. If the distance between the two masses (measured from center to the center) is now doubled, the gravitational forces becomes
   A. 1600 N B. 800 N C. 400 N D. 200 N E. 100 N

2. A ball of mass m attached to a string is moving in a horizontal circle of radius r with a uniform speed of v. The tension in the string (i.e. the force needed to keep the ball moving in a circle) is F_T. If the velocity of the ball decreases to v/3 (i.e. 1/3 its original velocity), what is the new tension in the string?
   A. $F_T/9$ B. $F_T/3$ C. F_T D. $3F_T$ E. $9F_T$

3. The acceleration of a free-falling object on some planet, does not depend on which of the following? A. The planet's mass B. The object's mass
   C. The distance of the object from the planet's center D. The Gravitational Constant

4. The term "astronomic unit" is defined as
   A. the average distance between the Earth and the Moon.
   B. the average diameter of the Moon's orbit about the Earth.
   C. the average distance between the Earth and the Sun.
   D. the average diameter of Earth's orbit about the Sun.
   E. the orbital period of Earth.

5. When an object experiences uniform circular motion, the direction of the acceleration is
   A. in the same direction as the velocity vector.
   B. in the opposite direction of the velocity vector.
   C. directed toward the center of the circular path.
   D. directed away from the center of the circular path.
   E. straight down towards the ground.

6. The orbital speed of a planet in our solar system does not depend upon
   A. Newton's gravitational constant G.
   B. the Sun's mass.
   C. the planet's mass.
   D. the planet's orbital radius

7. a. Based on the data in the table on the back of this test, which planets in our solar system have the longest orbital periods?

   Farther planets

   b. Choose one of Kepler's Laws and explain how it supports your answer to part A.

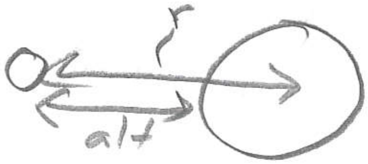
Applies to different satellites — 3rd Law: Period^2 is proportional to orbital radius^3

Applies to same satellite — 2nd Law → Farther = slower or equal area

5-6

8. Explain or show the difference between a satellite's orbital radius and its altitude.

3pts 1/2

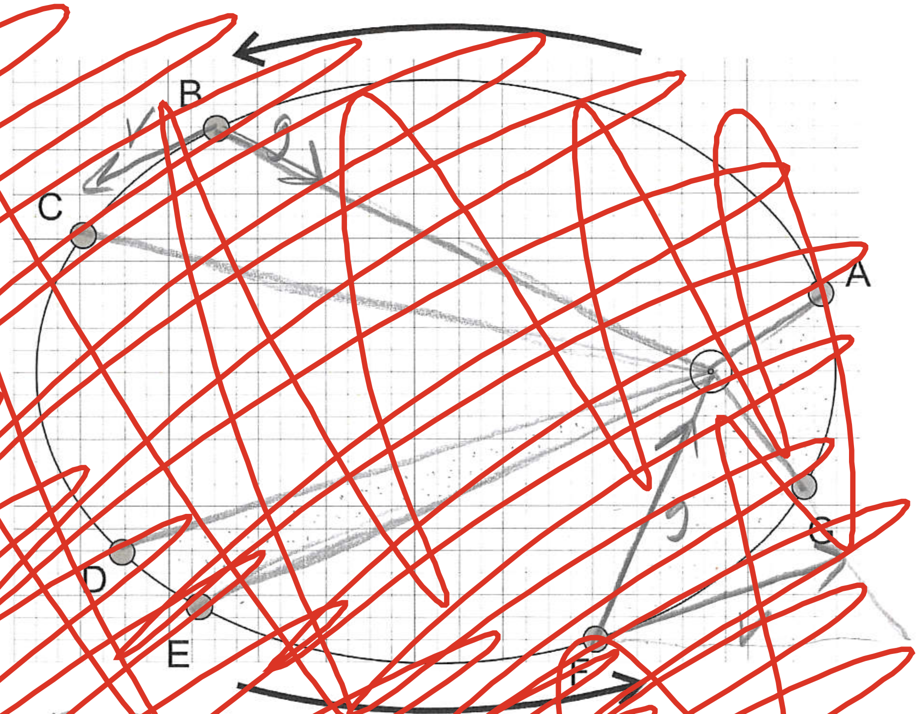


9. The diagram on the right shows the orbit of a planet around the sun. Between which two consecutive pairs of lettered points does the planet spend equal times? Circle the two pairs. Add graph

A&B B&C C&D D&E E&F F&G G&A

to A

1/2



10. Rank the lettered locations in order of the speed of the satellite at each location. List them from fastest to slowest.

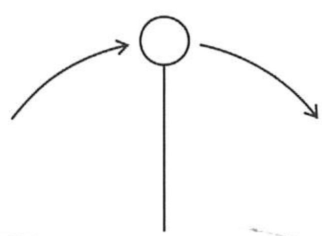
Fastest: A, G, F, D, E, B, C

11. At points **B and F** on the diagram, draw and label vectors for velocity and gravitational force. **Lengths must be proportional to magnitude.**

2pts 1/2 for each length + direction

Problems: 3pts Right Formulas - 1 Labeled Diagram Right Values - 1 work - 1

1. *[Hint: Read the entire question and pay close attention to the bold words.]* A playful lunar explorer swings a ball on a string. The 1kg ball is traveling in 0.5m radius vertical circles at a constant speed of 5m/s. The value of g on the moon is $1.63 m/s^2$. Give the **magnitude and direction** of the **net force** that is acting on the ball at the **top** of its swing.



$-1\frac{1}{2}$

$$\Sigma F = \frac{mv^2}{r} = \frac{1kg\,(5m/s)^2}{0.5m} = 50N \text{ Downward (center)}$$

$$\Sigma F = -\frac{mv^2}{r} = -mg - T$$

$$T = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} g\right) = 1kg\left(\frac{(5m/s)^2}{0.5m}\right)(1.63m/s^2) = 80N$$

2pts 1pt Σ

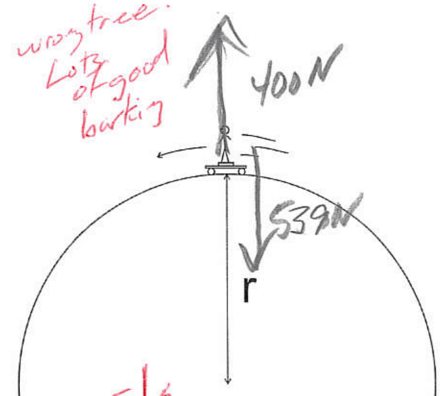
6

Formula for right thing but without enough given

$-\frac{1}{2}$ = wrong tree. Lots of good barking

2. A skateboarder stands on a bathroom scale on top of a skateboard as she travels over the top of a circular skate park feature. Her weight is 550N, and you may assume that her speed is momentarily constant at 8m/s. If the scale reads 400N at the top of the hill, what is the radius of the hill's curve?

55kg



$F_N - mg = -\frac{mv^2}{r}$

$\Sigma F = 400N - 539N = -55kg \frac{(8m/s)^2}{r}$

$r = 25.3m$

-1

$\frac{mv^2}{r} = -F_N - mg$

$\overset{400}{F_N} = \frac{mv^2}{r}$

-1

8.8m

3. A 40kg child is swinging on a massless swing in a vacuum. The child is swinging in arcs with a radius of 3m. At the lowest point in her swing, her speed is 3m/s. Assuming that her speed is constant in this part of her swing, what is the tension in the rope when she is at this lowest point?

$T - mg = \frac{mv^2}{r} \Rightarrow T = m\left(g + \frac{v^2}{r}\right)$

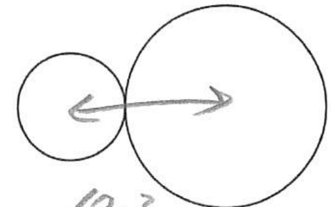
$= 40kg\left(9.8m/s^2 + \frac{(3m/s)^2}{3m}\right)$

$T = 512N$

-1

$T = 120$

4. One sphere has a radius of 0.1m, and the other sphere has a radius of 0.2m. They both have a mass of 0.7kg, and they are touching. Calculate the gravitational force between them.



$F_g = 6.67\times10^{-11}\left(\frac{(0.7kg)(0.7kg)}{0.3m^2}\right) = 3.6\times10^{-10}N$

-1/2

5. Use your knowledge of the Earth's orbit and the data at the back of this quiz to find the orbital period of Mars, in Earth years.

5/6

$A =$ Earth

$B =$ Mars

$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$

$\frac{1\,y^2}{T_{Mars}^2} = \left(\frac{1.496\times10^{11}m}{2.278\times10^{11}m}\right)^3$

$T_{Mars} = 1.88\,ey$

$\frac{1}{2}$ - units

7

6. A satellite orbits the Earth at an **altitude** of $2\text{x}10^6$m. Use the data on the back of this test to solve the following problems related to the satellite.

a. What is the satellite's orbital radius?

– minus 2 wait with answer

Orb r = alt + planet rad = $2\times10^6\,m + 6.37\times10^6\,m = 8.37\times10^6\,m$

b. What value of "g" is experienced by the satellite?

$$g = \frac{GM}{r^2} = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\left(\frac{5.979\times10^{24}\,kg}{(8.37\times10^6\,m)^2}\right) = 5.7\,m/s^2$$

7. Extraterrestrial explorers insert a satellite into a circular orbit around a newly discovered planet in a distant solar system. The satellite has a period of $1.20\text{x}10^5$seconds and an orbital radius of $5.60\text{x}10^7$m.

a. What is the speed of the satellite? left out → –1

$$v = \frac{d}{t} = \frac{2\pi(5.6\times10^7\,m)}{1.2\times10^5\,s} = 2.93\times10^3\,m/s$$

b. What is the mass of the planet around which the satellite orbits?

$$v = \sqrt{\frac{GM}{r}} \quad \frac{v^2 r}{G} = M = \frac{(2.93\times10^3\,m/s)^2(5.6\times10^7\,m)}{6.67\times10^{-11}\frac{Nm^2}{kg^2}} = 7.2\times10^{24}\,kg$$

Planetary Data

Name	Planetary Radius (meters)	Mass (kg)	Orbital Radius (meters)
Sun	696×10^6	1.991×10^{30}	-
Mercury	2.43×10^6	3.2×10^{23}	5.8×10^{10}
Venus	6.073×10^6	4.88×10^{24}	1.081×10^{11}
Earth	6.3713×10^6	5.979×10^{24}	1.4957×10^{11}
Mars	3.38×10^6	6.42×10^{23}	2.278×10^{11}

13/13

8

Physics 200

23-24 Test: Circles and Gravity

Name: Stapleton

Helpful Information:

$G = 6.67 \times 10^{-11} Nm^2/kg^2$ $M_{Earth} = 5.97 \times 10^{24}$ kg $M_{Moon} = 7.35 \times 10^{22}$ kg   Earth Radius = 6.378×10^6 m

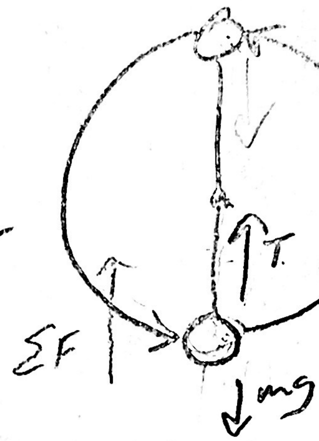
Earth Orbital Radius = 1.50×10^{11} m   Moon Radius = 1.74×10^6m

No units ok

1. A 0.2kg ball on a string is swinging in vertical circles with a radius of 0.3m. The ball's speed is constant at 4m/s.

   a. Where in the ball's path is string tension highest? Bottom

   b. What is the string tension at that point?



$$\Sigma F = \frac{mv^2}{r} = T - mg$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = 0.2kg\left(9.8 m/s^2 + \frac{(4 m/s)^2}{0.3m}\right)$$

$$\boxed{T = 12.6\,N}$$

3

10.67 8.71 -1

2. A rock is orbiting a planet in a stable, circular orbit with a constant speed of 800m/s. The rock's orbital radius is 30,000m. What is the mass of the planet that is being orbited?

$$\boxed{V = \sqrt{\frac{GM}{r}}}$$

$$800 m/s = \sqrt{\frac{6.67\times10^{-11} \frac{Nm^2}{kg^2}(M)}{30{,}000 m}}$$

$$M = 2.88\times10^{20} kg$$

mv^2

3

$$M = \frac{v^2 r}{G}$$

3. What is the force of gravitational attraction between the Earth and an astronaut orbiting the Earth at an orbital radius of 35,000m? The astronaut's mass is 65kg.

or $3.5\times10^7 m$

3

$$F_g = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\frac{(5.97\times10^{24} kg)(65kg)}{(3.5\times10^4 m)^2}$$

or $\times 10^7$

$$\boxed{F_g = G\frac{M_1 M_2}{r^2}}$$

Found F_g -2

$2.11\times10^7 N$

or

21.1N

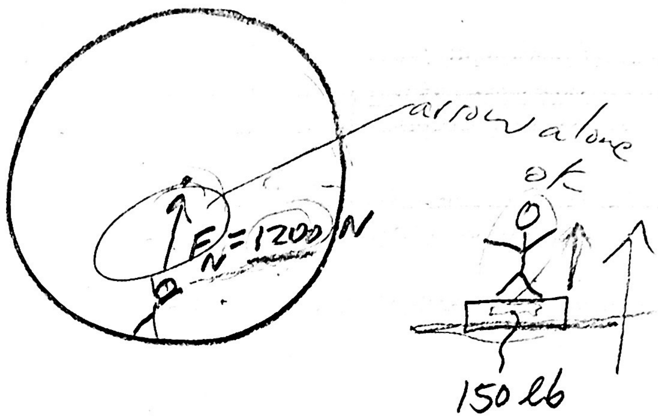
9

4. On a spacecraft traveling at a constant speed to planet X, "artificial gravity" is preparing the astronauts for the much *stronger* gravity of the new planet. The spacecraft creates this sensation of gravity by rotating, causing each astronaut to move at a constant speed of 15m/s (for simplicity, assume the astronauts stand still the whole time). One particular astronaut, who weighs 700N on Earth, experiences a sensation of weight 1,200N on the spacecraft.

4

a. Draw a diagram showing the following: the astronaut, the space station, the individual force(s) acting on the astronaut.

|



extra force(s) or wrong #s $-1/2$

b. What is the net force acting on the astronaut in your diagram? Give both magnitude and direction.

Magnitude of Net Force = $1/2$ 1200 N

Direction of Net force: $1/2$ Toward the center

$F_N = 500$ it

c. Calculate the radius of the astronaut's rotations.

1

$\Sigma F = \frac{mv^2}{r} = 1200N \Rightarrow \frac{71.4kg(15m/s)^2}{r} = 1200N$

$r = 13.4m$

32.14m

$w = mg$ $\quad 700N = m(9.8m/s^2)$

$m = 71.4kg$

5. The people of Earth have decided that gravity is too strong. We're too heavy, and we're tired of putting up with $g = 9.8m/s^2$. We want to lose some weight by reducing the value of g at Earth's surface to a more tolerable $8m/s^2$. How could we do this? Decribe the value(s) you would have to change – and what you would have to change the value(s) to -- in order to adjust g in this way.

Spin it faster

2 $\boxed{g = \frac{GM}{r^2}}$

1

$8m/s^2 = \frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}(5.97\times10^{24}kg)}{r^2}$

adjust r to this ($r = 7.05\times10^6m$)

$8m/s^2 = \frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}(M)}{(6.378\times10^6m)^2}$

adjust M to this ($M = 4.88\times10^{24}kg$)

$\uparrow F_N \downarrow mg$

$\frac{mv^2}{r} = m(9.8m/s^2) - m(8m/s^2)$

$v = 3,388m/s \rightarrow 3.3hr$ rot.