Practice with Conservation of Momentum - Answers

Conceptual Questions

1. Why is momentum conserved for ALL collision, regardless of whether they are elastic or not?
Newton's 3rd Law says that each object feels the same force, but in opposite directions. By extension, they both feel equal and opposite impulses so the change on momentum is equal and opposite. One might say that one's loss is the other's gain.

2. A Superball is dropped from a height h onto a hard steel plate (fixed to the Earth), from which it rebounds at very nearly its original speed. (a) Is the momentum of the ball conserved during any part of this process? (b) If we consider the ball and Earth as our system, during what parts of the process is momentum conserved?
(a) The momentum of the ball is not conserved during any part of the process, because there is an external force acting on the ball at all times – the force of gravity. And there is an upward force on the ball during the collision. So considering the ball as the system, there are always external forces on it, and so its momentum is not conserved.
(b) With this definition of the system, all of the forces are internal, and so the momentum of the Earth-ball system is conserved during the entire process.

3. At a hydroelectric power plant, water is directed at high speed against turbine blades on an axle that turns an electric generator. For maximum power generation, should the turbine blades be designed so that the water is brought to a dead stop, or so that the water rebounds?
Curved blades help the water rebound. Bouncing is a bigger change in momentum than just stopping.

Problems

1. A child in a boat throws a 6.40 kg package out horizontally with a speed of 10.0 m/s. Calculate the velocity of the boat immediately after, assuming that it was initially at rest. The mass of the child is 26.0 kg, and that of the boat is 45.0 kg. Ignore water resistance.

The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let "A represent the boat and child together, and let "B" represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B)v = m_A v_A' + m_B v_B' \quad \rightarrow$$
$$v_A' = -\frac{m_B v_B'}{m_A} = -\frac{(6.40\text{ kg})(10.0\text{ m/s})}{(26.0\text{ kg} + 45.0\text{ kg})} = \boxed{-0.901\text{ m/s}}$$

2. A 12,600-kg railroad car travels alone on a level frictionless track with a constant speed of 18.0m/s. A 5350-kg load, initially at rest, is dropped onto the car. What will be the car's new speed?

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = (m_A + m_B) v' \rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(12{,}600\ \text{kg})(18.0\ \text{m/s}) + 0}{(12{,}600\ \text{kg}) + (5350\ \text{kg})} = \boxed{12.6\ \text{m/s}}$$

3. A 3800 kg open railroad car coasts along level tracks with a constant speed of 8.60 m/s. Snow begins to fall vertically and fills the 3.50 kg/min. Ignoring friction with tracks, what is the speed of the car after 90 min?

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A = (m_A + m_B) v'_A$$

$$v'_A = \frac{m_A v_A}{m_A + m_B} = \frac{(3800\ \text{kg})(8.60\ \text{m/s})}{3800\ \text{kg} + \left(\frac{3.50\ \text{kg}}{\text{min}}\right)(90.0\ \text{min})} = 7.94\ \text{m/s} \approx \boxed{7.9\ \text{m/s}}$$

4. A softball of mass 0.220 kg that is moving with a speed of $8.5\ \text{m/s}$ collides head-on and elastically with another ball initially at rest. Afterward the incoming softball bounces backward with a speed of $3.7\ \text{m/s}$. Calculate (*a*) the velocity of the target ball after the collision, and (*b*) the mass of the target ball.

4. Let A represent the moving softball, and let B represent the ball initially at rest. The initial direction of the softball is the positive direction. We have $v_A = 8.5\ \text{m/s}$, $v_B = 0$, and $v'_A = -3.7\ \text{m/s}$.

(*a*) Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 8.5\ \text{m/s} - 0 - 3.7\ \text{m/s} = \boxed{4.8\ \text{m/s}}$$

(*b*) Use momentum conservation to solve for the mass of the target ball.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow$$

$$m_B = m_A \frac{(v_A - v'_A)}{(v'_B - v_B)} = (0.220\ \text{kg}) \frac{(8.5\ \text{m/s} - -3.7\ \text{m/s})}{4.8\ \text{m/s}} = \boxed{0.56\ \text{kg}}$$

5. Two bumper cars in an amusement park ride collide elastically as one approaches the other directly from the rear (Fig. 7–34). Car A has a mass of 450 kg and car B 550 kg, owing to differences in passenger mass. If car A approaches at $4.50\ \text{m/s}$ and car B is moving at $3.70\ \text{m/s}$, calculate (*a*) their velocities after the collision, and (*b*) the change in momentum of each.

5. Let the original direction of the cars be the positive direction. We have $v_A = 4.50\ \text{m/s}$ and $v_B = 3.70\ \text{m/s}$

(*a*) Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 0.80\ \text{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v_A' + m_B \left(0.80\,\text{m/s} + v_A'\right) \quad \rightarrow$$

$$v_A' = \frac{m_A v_A + m_B\left(v_B - 0.80\,\text{m/s}\right)}{m_A + m_B} = \frac{(450\text{ kg})(4.50\,\text{m/s}) + (550\text{ kg})(2.90\,\text{m/s})}{1000\text{ kg}} = \boxed{3.62\,\text{m/s}}$$ (*b*)

$$v_B' = 0.80\,\text{m/s} + v_A' = \boxed{4.42\,\text{m/s}}$$

Calculate $\Delta p = p' - p$ for each car.

$$\Delta p_A = m_A v_A' - m_A v_A = (450\text{ kg})(3.62\,\text{m/s} - 4.50\,\text{m/s}) = -3.96\times10^2\text{ kg}\cdot\text{m/s}$$

$$\approx \boxed{-4.0\times10^2\text{ kg}\cdot\text{m/s}}$$

$$\Delta p_B = m_B v_B' - m_B v_B = (550\text{ kg})(4.42\,\text{m/s} - 3.70\,\text{m/s}) = 3.96\times10^2\text{ kg}\cdot\text{m/s}$$

$$\approx \boxed{4.0\times10^2\text{ kg}\cdot\text{m/s}}$$

The two changes are equal and opposite because momentum was conserved.

6. A 0.280-kg croquet ball makes an elastic head-on collision with a second ball initially at rest. The second ball moves off with half the original speed of the first ball. (*a*) What is the mass of the second ball? (*b*) What fraction of the original kinetic energy $(\Delta\text{KE}/\text{KE})$ gets transferred to the second ball?

(*a*) Momentum will be conserved in one dimension. Call the direction of the first ball the positive direction. Let A represent the first ball, and B represent the second ball. We have $v_B = 0$ and $v_B' = \frac{1}{2}v_A$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v_A' - v_B'\right) \quad \rightarrow \quad v_A' = -\tfrac{1}{2}v_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow \quad m_A v_A = -\tfrac{1}{2}m_A v_A + m_B \tfrac{1}{2}v_A \quad \rightarrow$$

$$m_B = 3m_A = 3(0.280\text{ kg}) = \boxed{0.840\text{ kg}}$$

(*b*) The fraction of the kinetic energy given to the second ball is as follows.

$$\frac{KE_B'}{KE_A} = \frac{\frac{1}{2}m_B v_B'^2}{\frac{1}{2}m_A v_A^2} = \frac{3m_A\left(\frac{1}{2}v_A\right)^2}{m_A v_A^2} = \boxed{0.75}$$