## Notes – Standing Waves, Division of The Octave, and Fret Calculations (and one gratuitous formula)

1. Approximate speed of sound in air: $(331.4 + 0.6 T_C)$ m/s   **T_C means Celsius Temp.

2. When two or more waves arrive at the same point, the resulting wave is a __Sum__ of the waves. This is a phenomenon called __Superposition__. If the disturbance corresponds to a force, then the forces add. Whatever the disturbance, the resulting wave is a simple addition of the disturbances of the individual waves. That is, their amplitudes add.
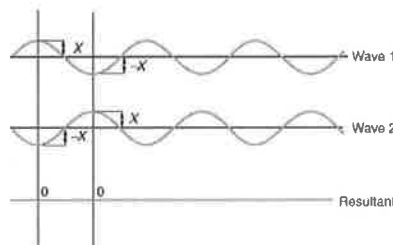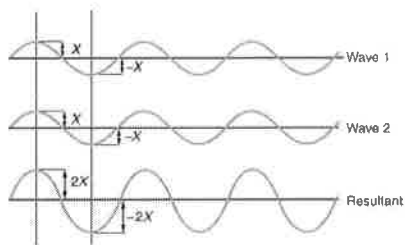
Pure __Constructive__ Interference   Pure __destructive__ Interference

3. When two __identical__ waves pass through each other moving in opposite directions, their disturbances add as they go by. If the two waves have the same __amplitude__ and __wavelength__, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place. This is called a __standing wave__.
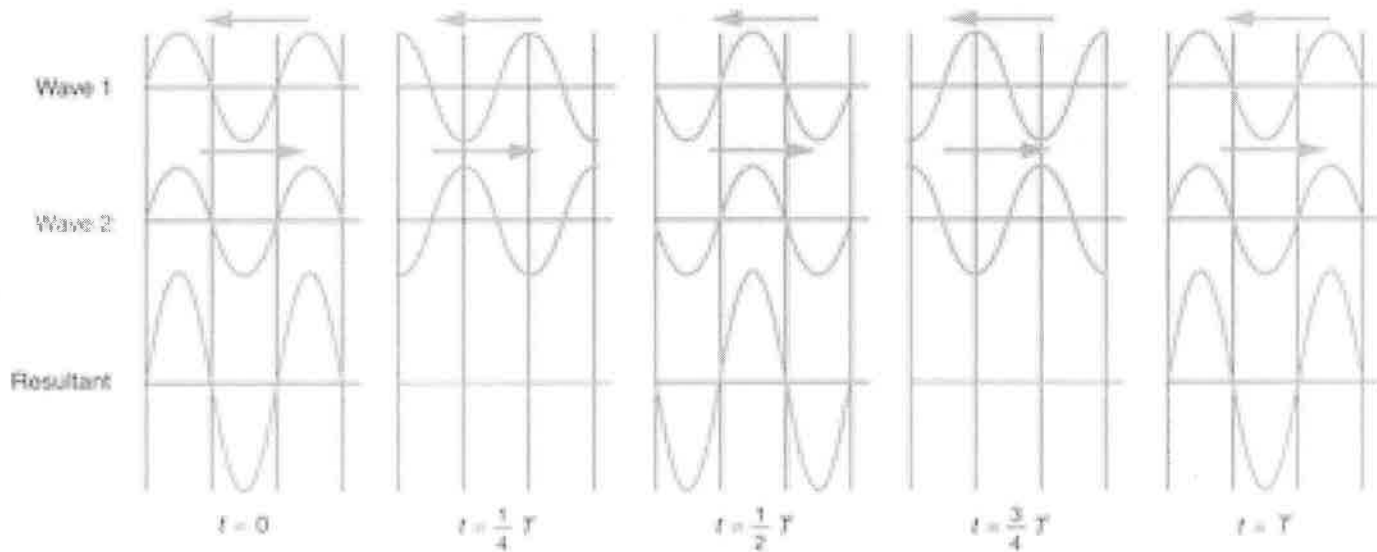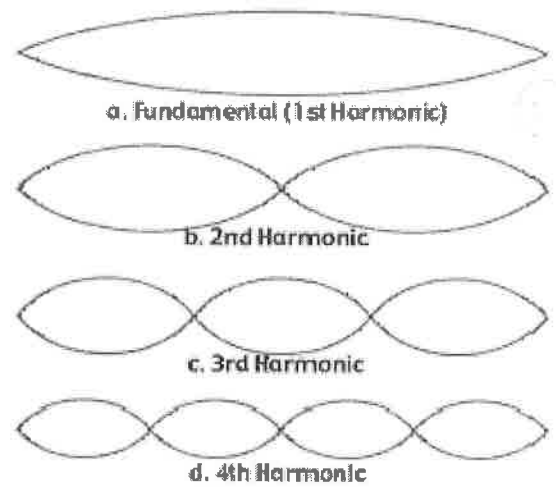
Wavelengths and Harmonics in Vibrating Strings fixed at both ends (e.g. a guitar string)

Nodes are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, because the string cannot move there. The locations of maximum amplitude in the standing wave are called **antinodes**.

—— See video

a. Fundamental (1st Harmonic)

b. 2nd Harmonic

c. 3rd Harmonic

d. 4th Harmonic

The vibrations of a string actually comprise several different wave patterns superimposed over one another. The loudest wave pattern is the **fundamental** (a.k.a. **1st harmonic**). These wave patterns are called **harmonics**, and they only occur at integer multiples of the fundamental frequency. For example, if the fundamental frequency is 10Hz, the 2nd harmonic would have a frequency of 20Hz; the 3rd harmonic would be 30Hz; the 4th harmonic = 40Hz. For wavelength, this relationship is inverted. The wavelength of the 2nd harmonic is $\frac{1}{2}$ the fundamental wavelength. The 3rd harmonic's wavelength is 1/3 the fundamental wavelength

4. For the figure on the top right, give the wavelength for each harmonic, in terms of the vibrating string length.

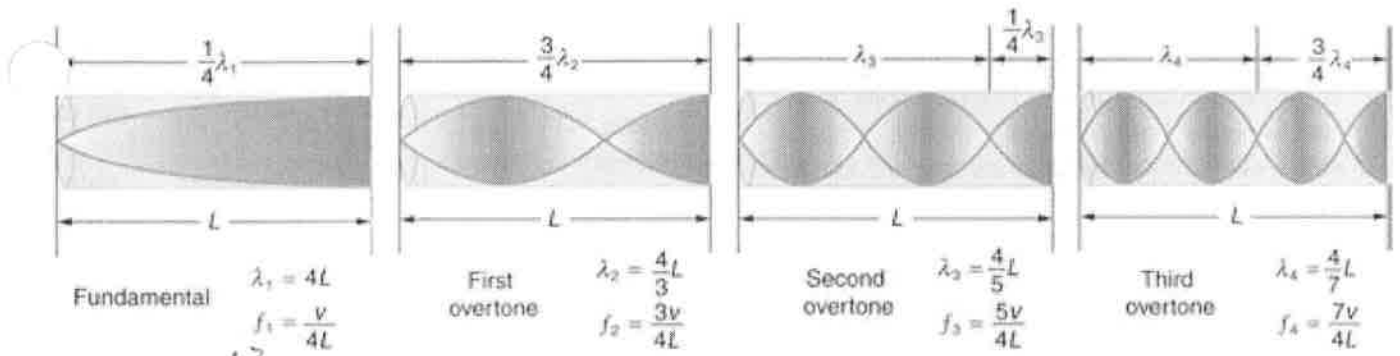Fundamental Wavelength = ____2____ string length $\lambda_{Fundamental} = 2L$

2nd harmonic Wavelength = ____1____ string length

3rd Harmonic Wavelength = ____$\frac{2}{3}$____ string length

4th Harmonic Wavelength = ____$\frac{1}{2}$____ string length

Wavelengths and Harmonics in a tube **open at one end** (e.g. an organ pipe)

Fundamental	First overtone	Second overtone	Third overtone
$\lambda_1 = 4L$	$\lambda_2 = \frac{4}{3}L$	$\lambda_3 = \frac{4}{5}L$	$\lambda_4 = \frac{4}{7}L$
$f_1 = \frac{v}{4L}$	$f_2 = \frac{3v}{4L}$	$f_3 = \frac{5v}{4L}$	$f_4 = \frac{7v}{4L}$

organ pip vid?
~overtones

5. How are waves in an organ pipe different than waves on a string?

- Harmonics are different, multiples of the fundamental.
- Air is moving, not a string
- Waves are longitudinal, not transverse

6. The diagram above represents the organ pipe waves as transverse waves. In reality, they are longitudinal. What is really happening to air molecules at the antinodes?

Air molecules move back and forth a _lot_ at the antinodes.

7. At the nodes, what are the air molecules doing?

Air molecules do not move at nodes.

8. For the fundamental, explain why there is a node at the left end and an antinode at the right?

End of pipe → prevents air movement

Node

Antinode

open pipe allows air to move back and forth
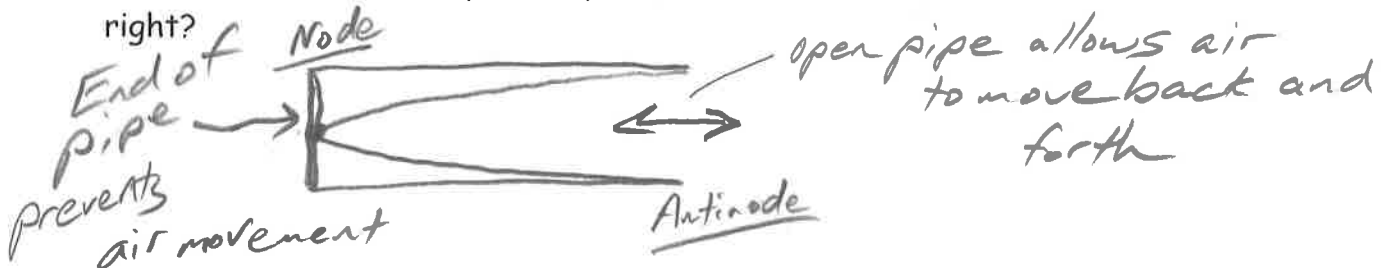
9. Draw the fundamental for a pipe that is closed at both ends. How much of a wavelength does the pipe length represent?

Node

Antinode

Node

$\lambda = 2L$
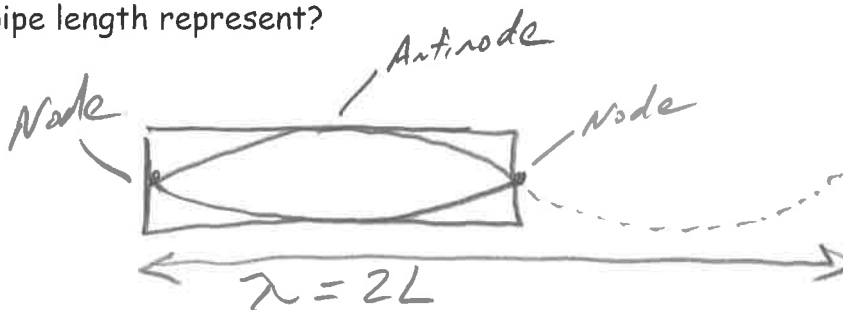
12 TET (12 Tone Equal Temperament) Division of the Octave

1. When musicians play a 1-octave scale, they play ___8___ notes. When we hear the musical notes at the bottom and top of a 1-octave scale, our ears perceive those notes as being the same notes, even though one sounds "higher" and one sounds "lower."

2. When two notes are separated by an octave, the higher note has a frequency that is

   ___twice___ the frequency of the

   lower note.

   For example, a musical note with a frequency of 110Hz is an A. If we start singing at that pitch and move gradually upward, we will reach the next A when we get to __220__ Hz. The next A after that will be heard at __440__ Hz.

3. In an 8 note, one octave scale, not every note on the instrument gets played. The music that most of us listen to actually divides each

   octave into __12__ equal parts. Each of these equal parts is called a

   ___half-step___. The musical system that divides an octave in this way is called

   12 Tone Equal
   Temperament

   This is the system that applies to most of the music that you have heard (probably).

Note Name	half steps up from starting note	Frequency (Hz)	Ratio: Current frequency / Previous frequency	Ratio of wavelength to starting note wavelength
A	0	440	NA	1
A# (or B♭)	1	466	1.059	0.944
B	2	494	1.059	0.891
C	3	523	1.059	0.841
C# (or D♭)	4	554	1.059	0.794
D	5	587	1.059	0.749
D# (or E♭)	6	622	1.059	0.707
E	7	659	1.059	0.667
F	8	698	1.059	0.630
F# (or G♭)	9	740	1.059	0.595
G	10	784	1.059	0.561
G# (or A♭)	11	831	1.059	0.530
A	12	880	1.059	0.5
A# (or B♭)	13	932	1.059	0.472
B	14	988	1.059	0.445
C	15	1047	1.059	0.420
C# (or D♭)	16	1109	1.059	0.397
D	17	1175	1.059	0.375
D# (or E♭)	18	1245	1.059	0.354
E	19	1319	1.059	0.334
F	20	1397	1.059	0.315
F# (or G♭)	21	1480	1.059	0.297
G	22	1568	1.059	0.281
G# (or A♭)	23	1661	1.059	0.265
A	24	1760	1.059	0.25

4. A one octave jump in pitch represents a ___doubling___ of sound wave frequency.
5. A two octave increase in pitch represents a $2^{(2)}$ increase in frequency.
6. A three octave increase in pitch represents a $2^{(3)}$ increase in frequency.
7. A four octave increase in pitch represents a $2^{(4)}$ increase in frequency.
8. A 1/12 octave increase in pitch (in other words, a half step) represents a $2^{(1/12)}$ increase in frequency. In other words, to raise the pitch of a sound by a half step its frequency must be multiplied by $2^{(1/12)} \approx 1.0595$.
9. To raise pitch by n half steps, one must multiply the current frequency by $2^{(n)}$.

10. $2^{(1/12)} \approx 1.0595$

String Instruments:

11. The frequency of sound produced by a string is affected by the string's __*length*__, __*tension*__, __*mass*__ and other characteristics.

12. The vibrating portion of a string extends from an instrument's __*bridge*__ to its __*nut*__.

13. Label the *nut, bridge, body, neck, and frets* on the string instrument to the right.
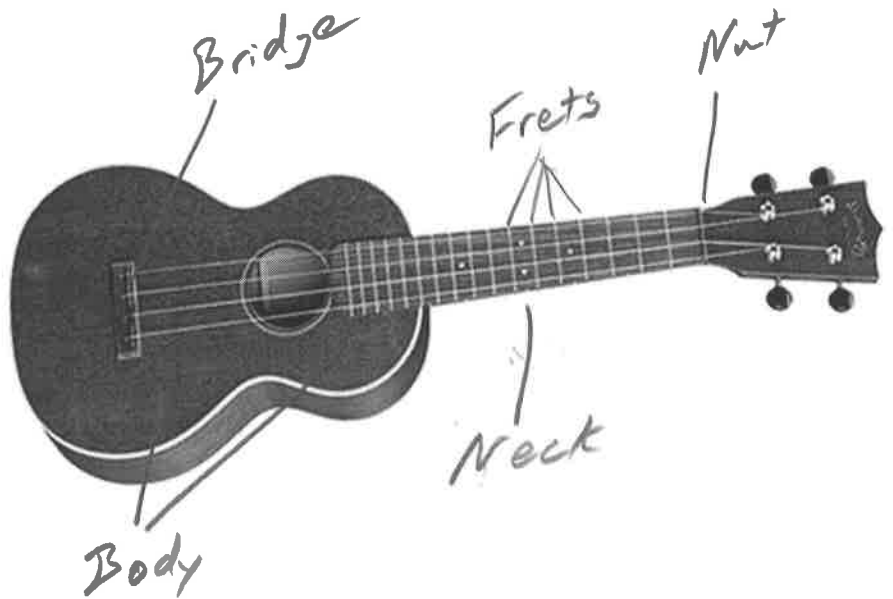
14. The purpose of frets is to allow the musician to precisely control *the length of the vibrating string, and therefore frequency/pitch.*

15. The purpose of the body is to *amplify sound by vibrating a larger surface that pushes more air.*

16. The purpose of the bridge is to *transfer the string's vibrations to the body, forcing the body to vibrate at the same frequency.*

Fret Placement:

When a string is plucked or bowed, many types of waves travel along it, producing a variety of standing waves. The dominant (loudest) standing wave is called the fundamental. There are also other harmonics (a.k.a. overtones), which have higher frequencies and pitch.

Nut   Bridge

a. Fundamental (1st Harmonic)

b. 2nd Harmonic

c. 3rd Harmonic

d. 4th Harmonic
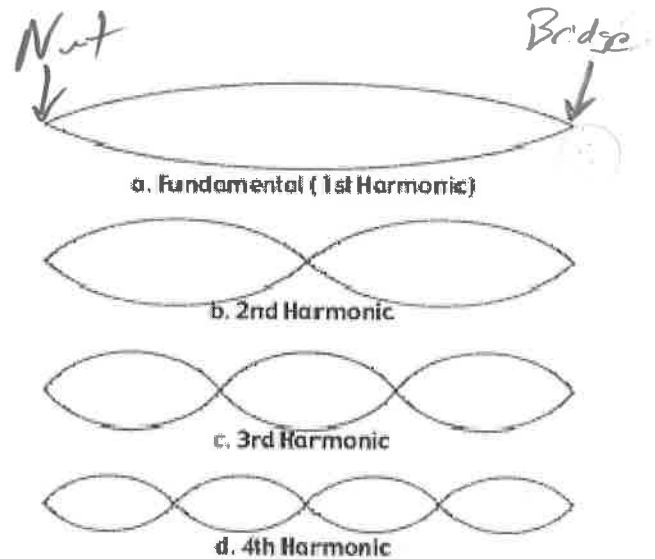
17. Suppose an instrument string is **50cm** long, and when the open string is plucked, its frequency is **400hz.**

   a. For purposes of tuning, we care about the fundamental vibration of the string. On the diagram to the right, label the position of the bridge and the nut. In this case, how many wavelengths does the vibrating string represent?

   $$L = \tfrac{1}{2}\lambda$$

   b. What is the full wavelength of the waves that are traveling down the string?

   $$\lambda = 2L = 2(50cm) = 100cm$$

   c. What is the relationship between string length and the wavelength of the string's fundamental standing wave?

   see b.

   d. What is the speed of those waves? Note: This speed is constant for a given string as long as the string's tension remains constant.

   $$v = \lambda f = 100cm \left(400Hz\right) = 4 \times 10^{4} \frac{cm}{s}$$

   e. The first fret (closest to the nut) on a finger board needs to correspond to a note that is one half-step higher than the open string. What is the frequency of a note one half step higher than the 400hz open string?

   $$f = f_0 \left(2^{\frac{\Delta halfsteps}{12}}\right) \qquad f = 400Hz\left(2^{\frac{1}{12}}\right) = 423Hz$$

   f. In order to produce that note, what wavelength must the string have? [hint: you know the string's wave speed]

   $$v = \lambda f \qquad 4 \times 10^{4} \frac{cm}{s} = \lambda\left(423Hz\right)$$
   $$\lambda = 94.6cm$$

   g. How long must the vibrating portion of the string be in order to produce that wavelength?

   $$\lambda = 2L \qquad 94.6cm = 2L$$
   $$L = 47.3cm$$

   h. How far from the nut should the first fret be located? In other words, by what distance must you shorten your string in order to raise your instrument's pitch by one half step?

   $$\text{Distance from nut} = L_0 - L = 50cm - 47.3cm = 2.7cm$$