

Making and Modeling Waves



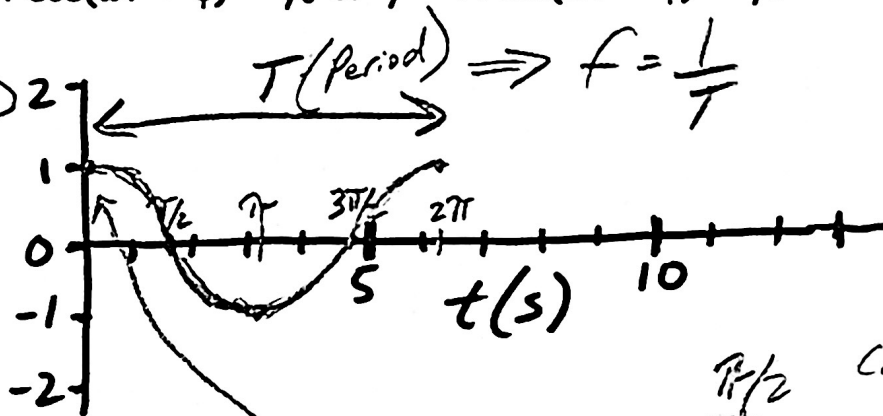
Waves are often made by objects that oscillate, and objects tend to oscillate in a predictable way. One type of oscillation is called **Simple Harmonic Motion** (or Simple Harmonic Oscillation). Some examples of objects that oscillate in this way are:

Pendulums (with small displacements), Instrument strings, Masses bouncing on springs, Vibrating rulers (that have just been bent and released)...

To model these waves, a modified sine curve is often used. One form of the **Wave Equation** looks like this...

$$y = A \cos(\omega t + \phi) + y_0 \text{ or } y = A \sin(\omega t + \phi) + y_0$$

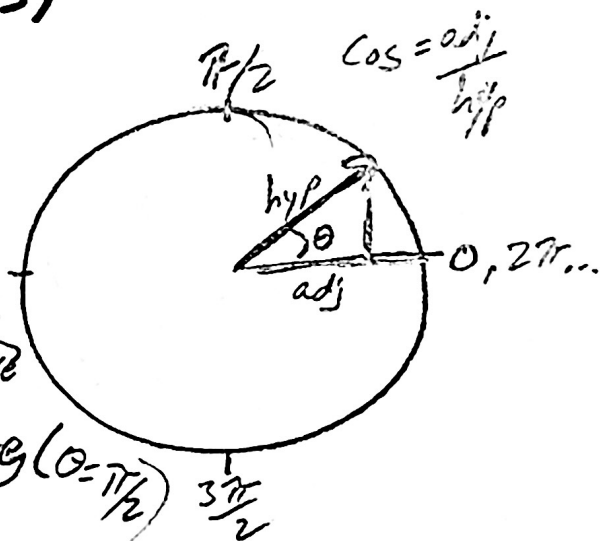
Sketch a graph of $y = \cos t$ $\cos(t)$, where $t = \text{time}$.



$$\omega = \frac{\Delta \theta}{t} = \frac{2\pi}{T} = \frac{2\pi}{1/2} = 2\pi f$$

$\phi = \text{Phase shift}$. Shifts starting position from $\theta = 0$ to $\theta = 0 + \phi$. Here to here ($\theta = \pi/2$)

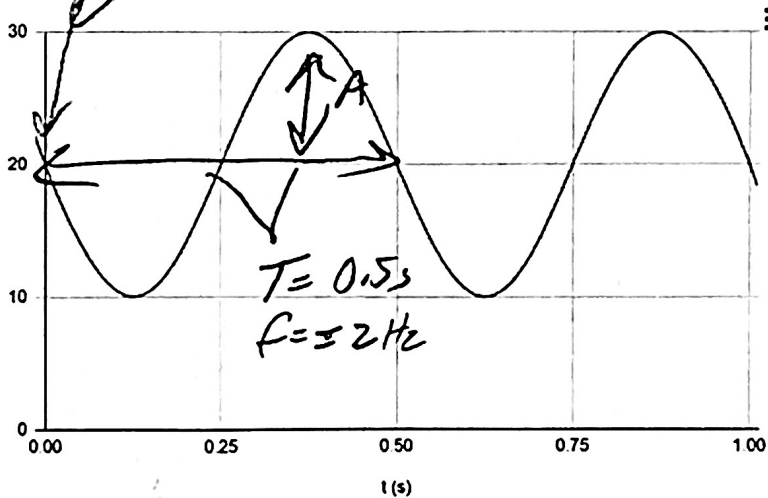
$y_0 = \text{Adj. Equilibrium height}$



Write an equation for the graph on the right, in the form

$$y = A \cos(\omega t + \phi) + y_0$$

$$y = 10 \text{ m } \cos(4\pi t + \frac{\pi}{2}) + 20 \text{ m}$$



$$y = A \cos(\omega t + \phi) + y_0$$

$$\omega = 2\pi f$$

$$\omega = \frac{2\pi}{T}$$



• Top right: $T = 4s \Rightarrow \omega = \frac{2\pi}{4} = \frac{\pi}{2}$

Starts here (trough)
So phase shift = π

$$A = 6m$$

Equilibrium $y = 2m = y_0$

$$y = 6 \cos\left(\frac{\pi t}{2} + \pi\right) + 2$$

• Bottom Left: $T = 0.25s \Rightarrow f = 4Hz \Rightarrow \omega = 2\pi(4) = 8\pi$

$$A = 3m \quad y_0 = -5m$$

$$\phi = 0$$

$$y = 3 \cos(8\pi t + 0) - 5$$

• Bottom Right: $T = 4s \Rightarrow \omega = \frac{2\pi}{4} = \frac{\pi}{2}$

$$A = 1.5m \quad y_0 = 0$$

$$\phi = \frac{3\pi}{2}$$

$$y = 1.5 \cos\left(\frac{\pi t}{2} + \frac{3\pi}{2}\right) + 0$$