Name: Key

Notes - 10.3 Dynamics of Rotational Motion: Rotational Inertia

1. A door opens more slowly if you push it too close to its hinges. The door will also open more slowly the more massive it is. From a torque and angular acceleration standpoint, the greater the applied force and the further it is applied from the pivot point (the rotational axis), the greater the angular acceleration. The angular acceleration is inversely proportional to mass. These relationships are analogous to the familiar relationships of force, mass, and acceleration.

2. Starting with Newton's 2nd Law, derive an expression for torque τ in terms of mass m, lever arm r and angular acceleration α.

$F = ma$

$F = m\alpha r$ $\quad a = \alpha r$

$rF = mr^2\alpha$ $\quad$ multiply by r

$\tau = mr^2\alpha$

$\tau = I\alpha$ $\quad I = mr^2$

3. Compare Newton's second law for linear motion and rotational motion.

$x \leftrightarrow \theta$

$v \leftrightarrow \omega$

$a \leftrightarrow \alpha$

$F \leftrightarrow \tau$

$m \leftrightarrow I$

$F = ma \leftrightarrow \tau = I\alpha$

4. The two definitions of torque: $\tau = rF$ $\quad \tau = I\alpha$

$\alpha = \frac{rF}{I}$ $\quad$ As $I \uparrow, \alpha \downarrow$ $\quad$ As $I \downarrow, \alpha \uparrow$

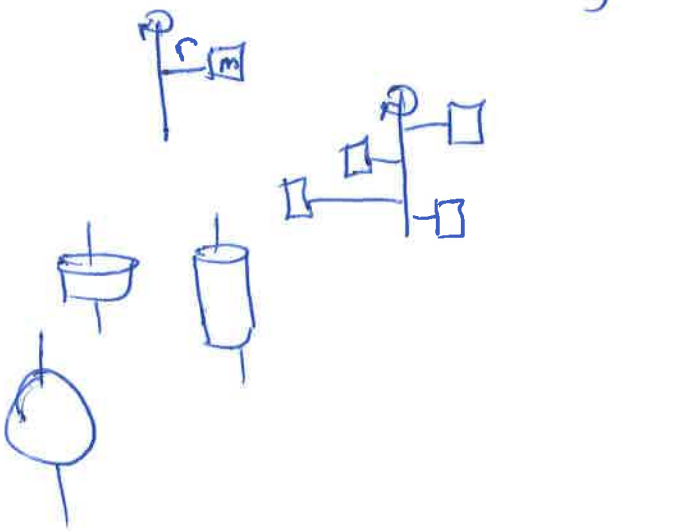
5. Rotational Inertia of Various Objects

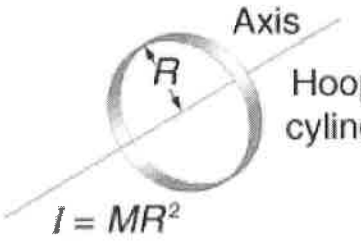
A. A single point mass: mr^2

B. Multiple point masses: $\sum_i m_i r_i^2$

C. Disk or cylinder: $\frac{1}{2}mr^2$

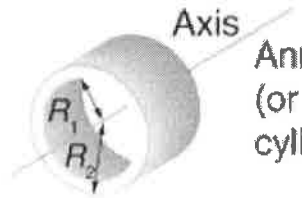
D. Solid sphere: $\frac{2}{5}mr^2$





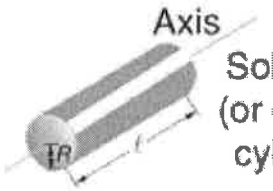
Hoop about cylinder axis

$I = MR^2$



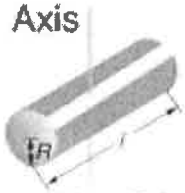
Annular cylinder (or ring) about cylinder axis

$I = \frac{M}{2}(R_1^2 + R_2^2)$



Solid cylinder (or disk) about cylinder axis

$I = \frac{MR^2}{2}$



Solid cylinder (or disk) about central diameter

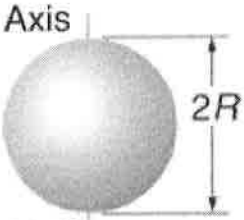
$I = \frac{MR^2}{4} + \frac{M\ell^2}{12}$

Thin rod about axis through center ⊥ to length

$I = \frac{M\ell^2}{12}$

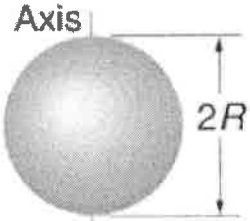
Thin rod about axis through one end ⊥ to length

$I = \frac{M\ell^2}{3}$



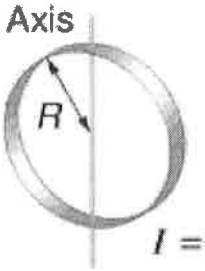
Solid sphere about any diameter

$I = \frac{2MR^2}{5}$



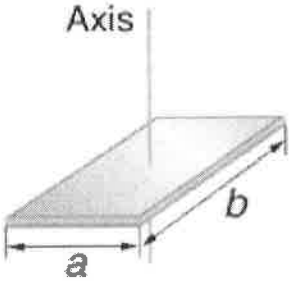
Thin spherical shell about any diameter

$I = \frac{2MR^2}{3}$



Hoop about any diameter

$I = \frac{MR^2}{2}$



Slab about ⊥ axis through center

$I = \frac{M(a^2 + b^2)}{12}$