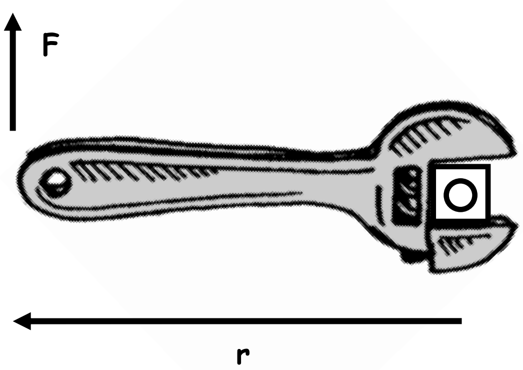
Unit 8 Packet (Physics 200): Rotational Motion Name: ___________

Torque Notes

I. τorque

A. The rotational equivalent of force is _____________. Its symbol is _______.



B. Torque = lever arm (r) x perpendicular force (F).

C. When θ = 90°,

When θ = 0°,

Torque is a maximum when θ = ___________.

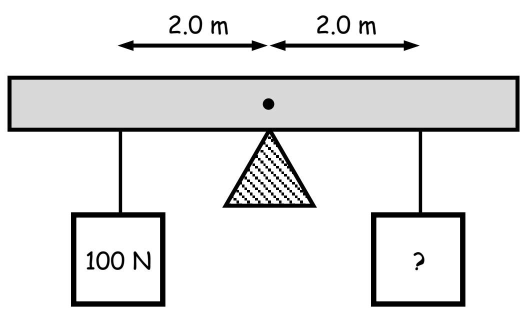
II. Rotational Equilibrium

A. In rotational equilibrium, $\sum_i \tau_i =$

In other words, the clockwise torques = the counterclockwise torques

B. Examples of rotational equilibrium:

1.



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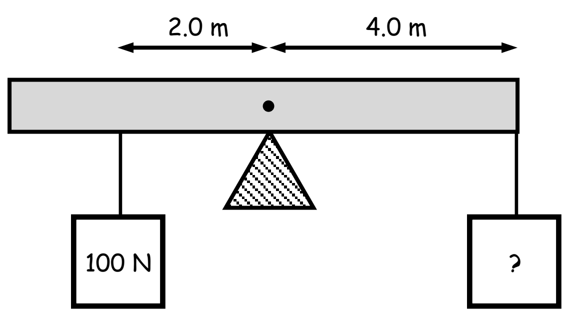

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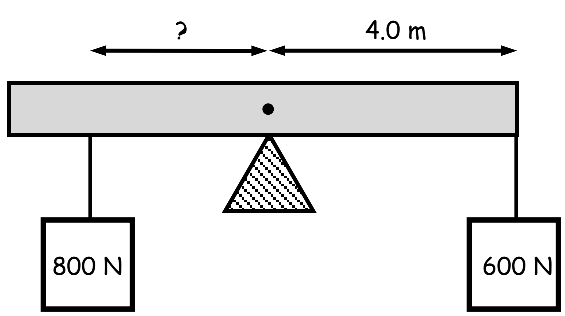
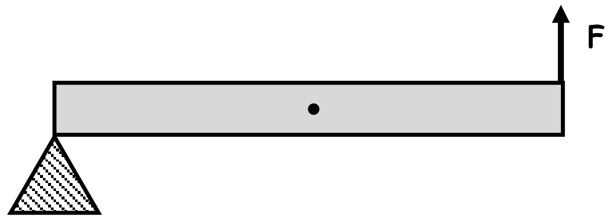
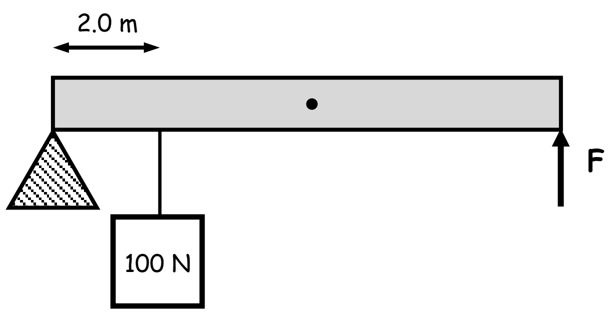


4. Find the force needed to hold the 5.0-meter beam that weighs 500 N level.



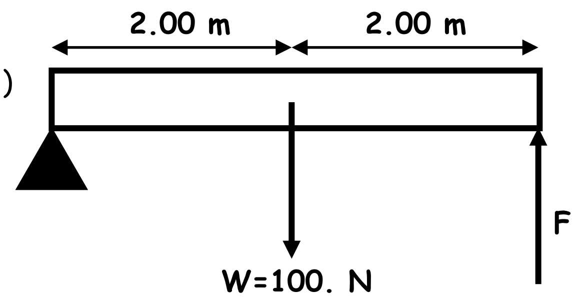
5. Find the force needed to hold the same beam level with the addition of a hanging weight.



Torque Practice

I. Find the force, F, needed to keep the bar level. The bar has a weight of 100. N. The location of the center of mass is designated by the downward arrow and the W.

A)



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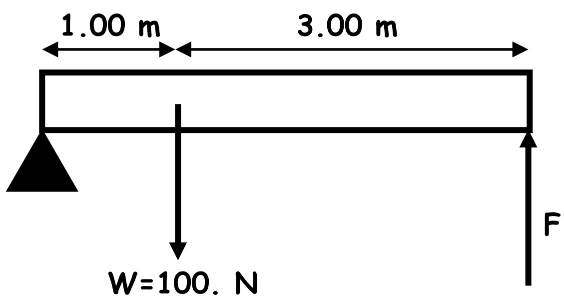
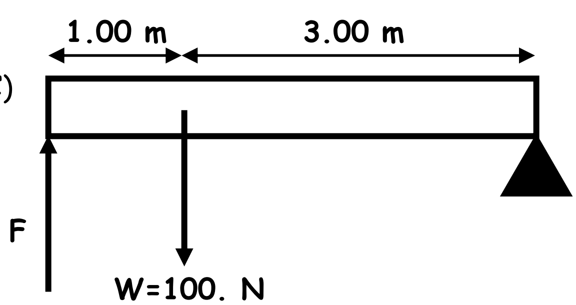
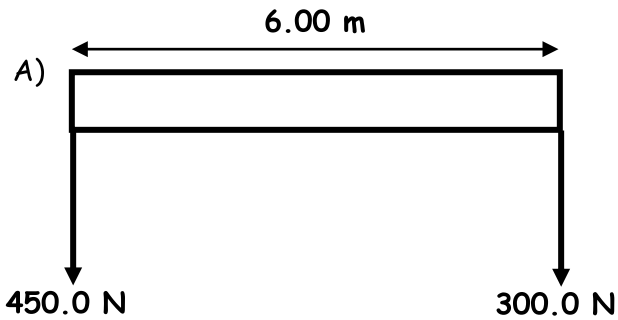


C)



II. Find the location of the fulcrum so that the bar balances. In case A), assume the bar has negligible mass. In cases B) and C), the location of the center of mass is again designated by the downward arrow and the W.

A)



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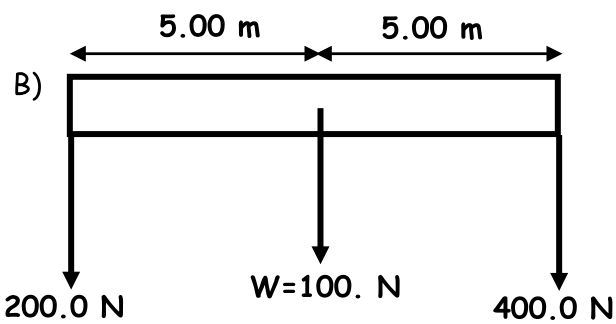

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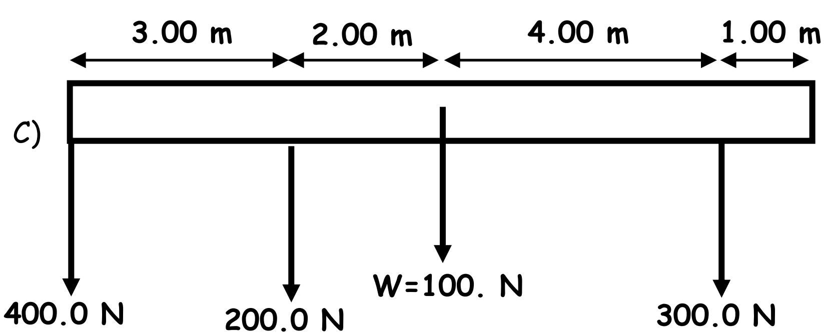
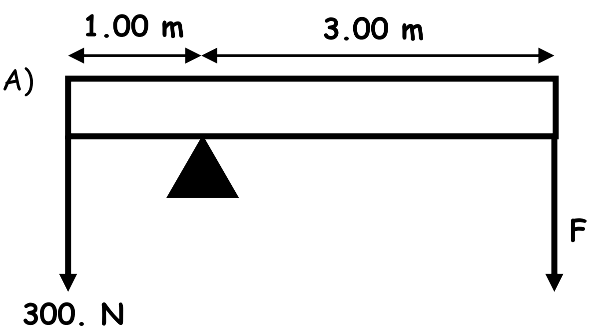


III. Find the force, F, needed to balance the bar. In cases A) and C), assume the bars have negligible mass. In case B), the location of the center of mass is again designated by the downward arrow and the W.

A)



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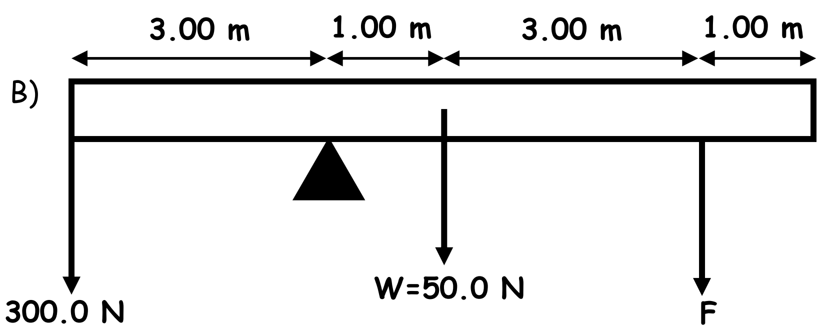
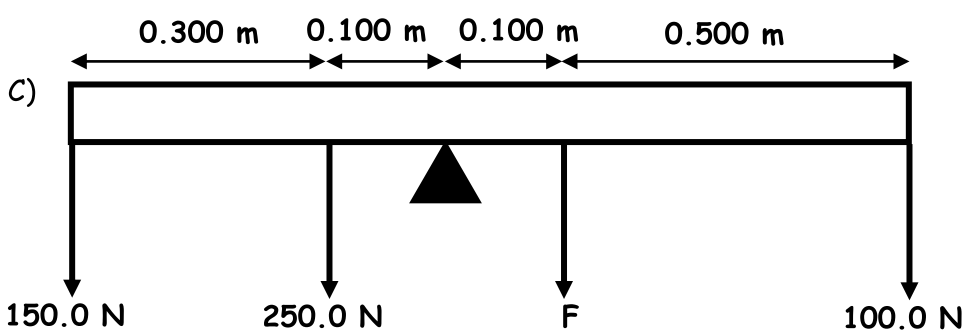


C)



Solutions:

I. A. 50.0 N B. 25.0 N C. 75.0 N

II. A. 2.40 m from left edge B. 1.43 m right of center C. 1.20 m left of center

III. A. 100 N B. 213 N C. 250 N

Notes – 10.1 Angular Acceleration

1. What is the definition of angular speed ω? What are the units of ω?

2. How are velocity and angular speed related?

3. What is the definition of angular acceleration α? What are the units of α?

4. Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s.

   A. Calculate the angular acceleration in rad/s^2. Show your work.

   B. If she now slams on the brakes, causing an angular acceleration of –87.3 rad/s^2, how long does it take the wheel to stop? Show your work.

5. How are tangential acceleration and angular acceleration related?

6. Distinguish between tangential acceleration (a_t) and centripetal acceleration (a_c)?

7. A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 km/h) in 4.20 s. What is the angular acceleration of its 0.320-m-radius wheels? Show your work.

Practice – 10.1 Angular Acceleration

1. At its peak, a tornado is 60.0 m in diameter and carries 500 km/h winds. What is its angular velocity in revolutions per second?

2. An ultracentrifuge accelerates from rest to 100,000 rpm in 2.00 min.
   A. What is its angular acceleration in rad/s^2?

   B. What is the tangential acceleration of a point 9.50 cm from the axis of rotation?

   C. What is the radial acceleration in m/s^2 and multiples of *g* of this point at full rpm?

3. A drum rotates around its central axis at an angular velocity of 12.60 rad/s. If the drum then slows at a constant rate of 4.20 rad/s^2,
   A. How much time does it take to come to a stop?
   [Note: $\omega_f = \omega_0 + \alpha t$]

   B. Through what angle does it rotate before coming to a stop?
   [Note: $\theta_f - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$]

4. Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what is the magnitude of the angular acceleration? [Note: $\theta_f - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$]

Answers:
1. 0.737 rev/s 2A. 87.3 rad/s^2 2B. 8.29 m/s^2 2C. 1.04×10^7 m/s^2, 1.06×10^6 g
3A. 3.00 s 3B. 18.9 rad 4. 2.0 rad/

Notes – 10.2 Kinematics of Rotation

1. Fill in the table below for the translational and rotation kinematic equations.

Translational (linear)	Rotational
$\Delta x = v\Delta t$ (or $v=\Delta x/\Delta t$)	
$v = v_0 + at$	
$\Delta x = v_0 t + \frac{1}{2}at^2$	
$v^2 = v_0^2 + 2a(\Delta x)$	

2. A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of 110 rad/s^2 for 2.00 s.

A. What is the final angular velocity of the reel? Show your work.

B. At what speed is fishing line leaving the reel after 2.00 s elapses? Show your work.

C. How many revolutions does the reel make? Show your work.

D. How many meters of fishing line come off the reel in this time? Show your work.

Practice – 10.2 Kinematics of Rotation

1. A spinning fishing reel has an initial angular velocity is ω_0 = 220 rad/s. If the fisherman applies a brake to the spinning reel, achieving an angular acceleration of –300 rad/s^2, how long does it take the reel to come to a stop?

2. Large freight trains accelerate very slowly. Suppose one such train accelerates from rest, giving its 0.350-m-radius wheels an angular acceleration of 0.250 rad/s^2.
   A. After the wheels have made 200 revolutions (assume no slippage), how far has the train moved down the track?

   B. After the wheels have made 200 revolutions (assume no slippage), what are the final angular velocity of the wheels and the linear velocity of the train?

Answers:
1. 0.733 s   2. A. 440 m   B. 25.1 rad/s, 8.77 m/s

Notes – 10.3 Dynamics of Rotational Motion: Rotational Inertia

1. Rotational Inertia (a.k.a. moment of inertia) is a rotational version of ______________. Whereas mass and ordinary inertia cause resistance to linear acceleration, an object's moment of inertia describes its resistance to ________________________________. The rotational inertia of an object depends both on its mass and the distance of that mass from the object's axis of rotation. As an example, consider a door. If the door's mass is increased, it will have a ____________________(higher,lower) resistance to rotational acceleration, and its moment of inertia will be ___________________ (higher,lower). If the door's mass is shifted "inward," so that it is closer to its axis of rotation, the door will have a ____________________ (higher,lower) resistance to rotational acceleration, and its moment of inertia will be ___________________ (higher,lower).

2. Starting with Newton's 2nd Law, derive an expression for torque τ in terms of mass m, lever arm r and angular acceleration α (and introduce I – "Rotational Inertia" or "moment of inertia")

3. Compare Newton's second law for linear motion and rotational motion.

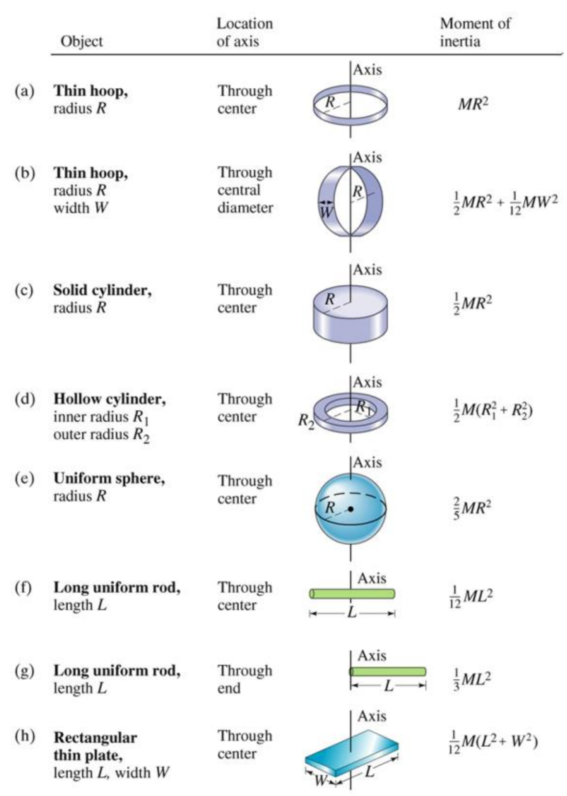
4. The two definitions of torque:

5. Rotational Inertia (I) of Various Objects

A. A single point mass:

B. Multiple point masses:

C. Other shapes – see chart



Practice 10.3: Rotational Dynamics

3. Calculate the rotational inertia of a solid sphere of mass M = 5.0 kg and a radius of R = 0.25 m.

4. Calculate the rotational inertia of a solid cylinder of mass M = 2.0 kg and a radius of R = 0.075 m about its central axis.

5. Suppose you exert a force of 180 N tangential to a 0.280-m-radius 75.0-kg grindstone (a solid disk).
   A. What torque is exerted?

   B. What is the angular acceleration assuming negligible opposing friction?

   C. What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?

Answers:
1. 0.733 s 2. A. 440 m B. 25.1 rad/s, 8.77 m/s 3. 0.13 kg m^2 4. 5.6 x 10^{-3} kg m^2
5. A. 50.4 N·m B. 17.1 rad/s^2 C. 17.0 rad/s^2

Notes – 10.4 Rotational Kinetic Energy

1. Starting with the linear (or tangential) kinetic energy formula, derive a formula for the rotational kinetic energy of a single mass m, with a velocity v, revolving around an axis at a radius r. The formula should be in terms of I and ω.

2. Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

4. Calculate the final speed of a hoop of the same radius (4cm) that is allowed to roll down an incline of the same height (2m)

5. Compare the speeds of thin hoops and solid cylinders, in general, after rolling down ramps (assuming the objects' radii and the ramp heights are identical, and that there is no friction).

Practice – 10.4 Rotational Kinetic Energy

1. What is the final velocity of a 1.00 kg hoop starting from rest that rolls without slipping down a hill 5.00 meters high?

2. What is the final velocity of a 1.0 kg solid disk/cylinder starting from rest that rolls without slipping down a hill 5.00 meters high?

3. Calculate the rotational kinetic energy of Earth on its axis. Assume the Earth is a uniform solid sphere of mass $M = 5.97 \times 10^{24}$ kg and a radius $R = 6371$ km.

4. What is the rotational kinetic energy of Earth in its orbit around the Sun? $M = 5.97 \times 10^{24}$ kg and $R = 150$ million kilometers.

5. A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping. Treating the ball as a spherical shell, calculate the vertical height it reaches.

Answers:
1. 7.00 m/s 2. 8.08 m/s 3. 2.56×10^{29} J 4. 2.66×10^{33} J 5. 5.44 m

Notes – 10.5 Angular Momentum and Its Conservation

1. Write the equation for linear momentum.

2. Write the equation for angular momentum.

3. State the Law of Conservation of Angular Momentum in words.

4. Write the equation for the Conservation of Momentum.

6. Suppose an ice skater is spinning at 0.800 rev/ s with her arms extended. She has a moment of inertia of 2.34 kg·m^2 with her arms extended and a moment of inertia equal to 0.363 kg·m^2 with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.)

A. What is her initial angular velocity, in rad/s?

B. What is her initial angular momentum?

C. What is her final angular velocity?

B. What is her rotational kinetic energy before and after she does this? Why does her KE_R change?

Practice – 10.5 Angular Momentum and Its Conservation

1. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after an initially motionless 22.0-kg child gets onto it by grabbing its outer edge? This might be easier to visualize if you picture the merry-go-round snagging the child and yanking him/her into motion. [You may assume that the merry-go-round is a uniform disc with $I=1/2\ mr^2$ and that the child is a point source with $I=mr^2$.]

2. Ice Skater

A. Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is 0.400 $kg \cdot m^2$.

B. He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s.

C. Suppose instead he keeps his arms in and allows friction of the ice to slow him from 6.00 rev/s to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?

3. What is the angular momentum of Earth rotating on its axis? $M_{Earth} = 5.97 \times 10^{24}$ kg and $R_{Earth} = 6371$ km. Assume the Earth is a solid uniform sphere ($I = 2/5\ mr^2$)

4. What is the angular momentum of the Moon in its orbit around Earth? The orbital radius of the Moon is 384,399 km, the Moon's mass is 7.35×10^{22} kg and its orbital period is 27.321 days. A) What value should you use for I? b) What is the angular momentum?

Answers:

1. 2.30 rad/s 2. A. 15.1 kg m^2/s B. 1.92 kg m^2 C. -0.503 N m
3. 7.05×10^{33} kg m^2/s 4. 2.89×10^{34} kg m^2/s

Pennington 2015-16 Chapter 9-10 Test Review and Practice Test

Key Equations

$\Delta x = \Delta\theta r$ $v = \omega r$ $a_t = \alpha r$ $a_r = a_c = \frac{v^2}{r} = \omega^2 r$

$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$

$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_i^2 + 2\alpha(\Delta\theta)$ $\omega = \omega_i + \alpha t$

$K_{rot} = \frac{1}{2}I\omega^2$ $I = \sum_{i=1}^{n} m_i r_i^2$ (discrete masses)

$\tau = rF = I\alpha$ $L = rp = rmv = I\omega$ When $\tau_{net}=0$, $L_i = L_f$ and $I_i\omega_i = I_f\omega_f$

For equilibrium, with regard to both rotational and linear motion: $\tau_{cw} = \tau_{ccw}$ and $\vec{F}_{net} = 0$

Multiple Choice

- Balancing beams
- Relationship of τ to r, F, I and α
- Units for all the parameters
- Ways to increase (or decrease) torque
- Use of kinematic equations to determine s, t, ω, α, θ
- Relationship of linear parameters s, v, a_t, a_r to rotational parameters θ, ω, α
- Which shape wins the race down a ramp
- Determining angular momentum
- Which configuration has more rotational inertia
- Definition of static equilibrium

Problems

- Taken from practice problems in class

Chapter 9-10 Practice Test

Part I. Multiple Choice

1. With the same non-zero clockwise torque applied, if an object's rotational inertia is increased, its angular acceleration
   A. increases. B. decreases. C. stays the same.

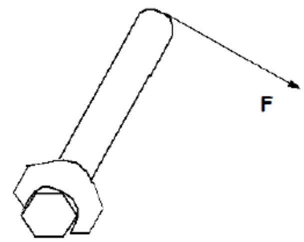
2. The torque applied to a bolt that is stuck can be increased by
   A. increasing the length of the lever arm.
   B. increasing the magnitude of the applied force.
   C. changing the direction of the force to be perpendicular to the lever arm.
   D. All of the above.

3. The units of angular speed are
   A. kg m^2 B. rad C. rad/s D. rad/s^2 E. s^{-1}

4. The units of angular acceleration are
   A. kg m^2 B. rad C. rad/s D. rad/s^2 E. s^{-1}

5. A wheel with a radius of 0.500 m rotates through an angle of 4π. How far has a point on the outer rim traveled?
   A. 0.500 m B. 3.14m C. 6.28 m D. 9.42 m E. 25.1 m

6. In an effort to tighten a bolt, a force **F** is applied as shown in the figure below. If the distance from the end of the wrench to the center of the bolt is 25.0 cm and F = 5.00 N, what is the magnitude of the torque produced by **F**?
   A. 0.00 N·m
   B. 1.25 N·m
   C. 5.00 N·m
   D. 75.0 N·m
   E. 125.0 N·m

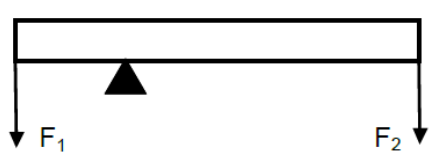


7. If a wheel turning at a constant rate completes exactly 100 revolutions in 10.0 s, its angular speed is:
   A. 0.314 rad/s B. 0.628 rad/s C. 10.0 rad/s D. 62.8 rad/s E. 314 rad/s

8. A child initially standing on the edge of a freely spinning merry-go-round moves to the center. Which one of the following statements is necessarily true concerning this event and why?
   A. The angular speed of the system increases because the moment of inertia of the system has decreased.
   B. The angular speed of the system decreases because the moment of inertia of the system has decreased.
   C. The angular speed of the system increases because the moment of inertia of the system has increased.
   D. The angular speed of the system decreases because the moment of inertia of the system has increased.
   E. The angular speed of the system remains the same because the net torque on the merry-go-round is zero.

9. When a spinning ice skater draws in her outstretched arms, her kinetic energy increases. Where does this added energy come from?
   A. The added energy comes from a decrease in her angular momentum.
   B. The added energy comes from the dark energy of the vacuum.
   C. The added energy comes from the work the skater does pulling in her arms.
   D. The added energy comes from the decreased angular speed.
   E. The added energy is transferred from the ice to the skater.

10. In order to increase the torque created by F_1 below, the fulcrum should be moved
   A. closer to F_1 B. closer to F_2



11. Which one of the following statements provides the best definition of rotational inertia?
   A. Rotational inertia is the momentum of a rotating object.
   B. Rotational inertia is equal to the mass of the rotating object.
   C. Rotational inertia is the resistance of an object to a change in its angular velocity.
   D. Rotational inertia is the resistance of an object to a change in its linear velocity.

12. For a body to be in equilibrium, what conditions must apply?
    A. $\tau_{cw} = 0$ and $\tau_{ccw} = 0$
    B. $F_{x\ net} = F_{y\ net}$
    C. $\tau_{cw} = \tau_{ccw}$ and $F_{x\ net} = F_{y\ net}$
    D. $\tau_{cw} = \tau_{ccw}$, $F_{x\ net} = 0$ and $F_{y\ net} = 0$

13. A comet orbiting the Sun can be considered an isolated system with no outside forces or torques acting on it. As the comet moves in its highly elliptical orbit, what remains constant?
    A. Its distant from the Sun
    B. Its angular speed
    C. Its linear speed
    D. Its angular momentum
    E. The gravitational force between the comet and the Sun.

14. A uniform disk, a hoop and a uniform solid sphere are held stationary at the top of a ramp. All 3 objects have the same mass and radius. When released, they roll down the ramp without slipping. Rank the objects according to their speed at the bottom of the incline from least to greatest.

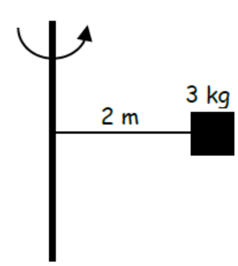
$I_{disk} = 1/2MR^2$ $\quad I_{hoop} = MR^2$ $\quad I_{solid\ sphere} = 2/5MR^2$

    A. hoop, disk, sphere
    B. disk, hoop, sphere
    C. sphere, hoop, disk
    D. sphere, disk, hoop

15. A disk initially rolls along the flat ground at a constant speed without slipping. If linear speed of the disk is now doubled,
    A. the angular speed is increases by 2X and the kinetic energy increases by 2X.
    B. the angular speed is increases by 4X and the kinetic energy increases by 2X.
    C. the angular speed is increases by 2X and the kinetic energy increases by 4X.
    D. the angular speed is increases by 4X and the kinetic energy increases by 4X.
    E. both the angular speed and the kinetic energy remain the same.

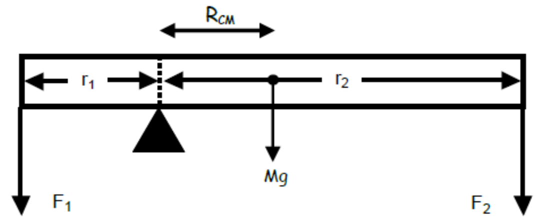
16. What happens when a spinning ice skater draws in her outstretched arms?
    A. Her moment of inertia decreases causing her to speed up.
    B. Her angular momentum decreases.
    C. The torque that she exerts increases her moment of inertia.
    D. Her angular momentum increases.
    E. Her moment of inertia decreases causing her to slow down.

17. A 3 kilogram mass is attached 2 meters from a rotating rod. What is the rotational inertia of the mass?
    A. 12 $kg \cdot m^2$
    B. 54 $kg \cdot m^2$
    C. 6 $kg \cdot m^2$
    D. 18 $kg \cdot m^2$
    E. 36 $kg \cdot m^2$



18. A torque applied to a solid object that is free to move will produce
    A. a linear acceleration.
    B. rotational equilibrium.
    C. an angular acceleration.
    D. rotational inertia.

19. Assume this bar has mass M and forces F_1 and F_2 are pulling down on each end. In order for the bar to be balanced:



A. $F_1 = F_2$
B. $F_1 < F_2$
C. $r_1 F_1 = r_2 F_2$
D. $\tau_{cw} = 0$ and $\tau_{ccw} = 0$
E. $r_1 F_1 = r_{CM} Mg + r_2 F_2$

II. Problems:

1. An ultracentrifuge accelerates from rest to a top speed of 1870 revolutions per second in 2.00 min.
   A. What is its angular acceleration in rad/s^2?
   B. What is the tangential acceleration at a point 7.00 cm from the axis of rotation?
   C. What is the radial (i.e. centripetal) acceleration at a point 7.00 cm from the axis of rotation when the centrifuge is rotating at top speed?

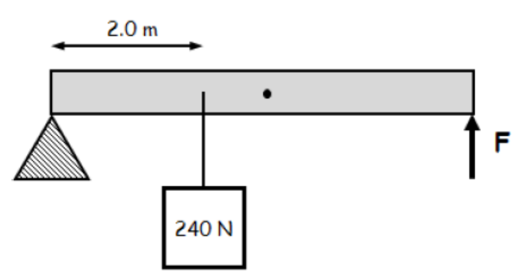
2. A drum rotates around its central axis at an initial angular speed of 11.50 rad/s. The drum then slows at a constant rate of 2.60 rad/s^2.
   A. How much time does it take to come to a stop?
   B. Through what angle does it rotate before coming to a stop?

3. Suppose you exert a 1450 N force tangent to the surface of a solid sphere that has a radius of 0.340 meters and a mass of 13.0 kg. $I_{solid\ sphere} = 2/5\ MR^2$.
   A. What torque is exerted?
   B. What is the angular acceleration assuming negligible opposing friction?

4. Ice Skater
   A. Calculate the angular momentum of an ice skater spinning at 3.60 rev/s given that her moment of inertia is 0.320 kg·m^2.
   B. She then reduces her rate of spin (her angular velocity) by extending her arms. Find the new value of her moment of inertia if her angular velocity decreases to 1.20 rev/s.

5. A 0.440-kg soccer ball with a radius of 11.2 cm is initially at rest at a height of 3.90 meters at the top of a ramp. It is then released. The soccer ball can be thought of as a hollow spherical shell, $I = \frac{2}{3}MR^2$.
   A. What is the final velocity of the soccer ball at the bottom of the ramp if the ramp is frictionless, so the ball just slides down the ramp without rolling?
   B. What is the final velocity of the soccer ball at the bottom of the ramp if there is friction and it rolls down the ramp without slipping?

6. What force needs to be applied at the end of a uniform 5.0-meter beam to keep it level. A 240 N mass is hung 2.0 meters from the fulcrum. The beam weighs 600 N.



4-Minute Drill
Chapter 9-10

Distance a point on a body moves as the body rotates through an angle θ

Velocity of a point on a body as the body rotates with angular speed ω

Acceleration of a point on a body as the body's rotation rate increases

Angular velocity in terms of θ

Angular acceleration in terms of ω

One of the rotational kinematic equations ($\Delta\theta =$)

Another rotational kinematic equation ($\omega=$)

One more rotational kinematic equation ($\omega^2=$)

Rotational kinetic energy formula

Total kinetic energy of a rolling body

Rotational inertia of discrete particle of mass m at a distance r from the axis

Rotational inertia of a cylinder with the axis through the center of the flat face

Rotational inertia of a solid sphere with the axis through the center

Torque in terms of force applied at a given distance from the rotational axis

Torque (Newton's 2nd Law for rotation)

Angular momentum

Another expression for angular momentum

Conservation of angular momentum