Physics 200 Name: ______________________

Momentum & Impulse: wrap-up and review

Coefficient of Restitution: tells how elastic a collision is; is a ratio of the separation speed of objects after a collision to their approach speed before the collision.

Coefficient of Restitution = $\frac{Separation\ Speed}{Closing\ speed}$

When e =1...

- objects separate as fast as they came together
- collision is **perfectly elastic.**
- No kinetic energy is lost.
- Example: A perfectly bouncy ball approaches the ground at 2m/s (closing speed) and then bounces back up with a speed of 2m/s (separation speed). e = 2/2

When e=0...

- objects do not separate
- the collision is **perfectly inelastic.**
- Kinetic energy is lost to friction.
- Example: a bullet approaches a ballistic pendulum at 500m/s (closing speed) and the bullet and pendulum then swing upward together (separation speed = 0; no separation). e =0/2

Coefficient of Restitution Formula = $e = \frac{V_b{}' - V_a{}'}{V_a - V_b}$ ...where Va = initial velocity of object A, Vb = initial velocity of object B, and Va' and Vb' = their final velocities.

When e=1, $V_b{}' - V_a{}' = V_a - V_b$

Example Problem:

Cart A has a mass of 4kg and an initial velocity of -2m/s. Cart B has a mass of 3kg and an initial velocity of 0m/s. If the carts collide with perfect elasticity (e=1), what are the carts' velocities after the collision?

$V_B{}' - V_A{}' = V_A - V_B \qquad V_B{}' - V_A{}' = -2m/s - 0m/s = -2m/s$

$V_B{}' = V_A{}' - 2m/s$

Using Cons of P:

$4kg(-2m/s) + 3kg(0m/s) = 4kg(V_A{}') + 3kg(V_B{}')$

$-8kg\,m/s = 4kg\,V_A{}' + 3kg(V_A{}' - 2m/s)$

$-8kg\,m/s = 4kg\,V_A{}' + 3kg\,V_A{}' - 6\,kg\,m/s$

$-2kg\,m/s = 7kg\,V_A{}'$

$V_A{}' = -0.29m/s$

$V_B{}' = -0.29m/s - 2m/s = -2.29m/s$

Problems

1. What is the magnitude of the momentum of a 28-g sparrow flying with a speed of 8.4 m/s?

$p = mv = (.028\,kg)(8.4\,\frac{m}{s}) = 0.235\,kg\,m/s$

2. A constant friction force of 25 N acts on a 65-kg skier for 20 s. What is the skier's change in velocity?

$F\Delta t = m\Delta v$

$(25)(20) = 65\,\Delta v$

$\Delta v = 7.69\,m/s$

3. A 0.145-kg baseball pitched at 39.0 m/s is hit on a horizontal line drive straight back toward the pitcher at 52.0 m/s. If the contact time between bat and ball is 3.00×10^{-3}s, calculate the average force between the ball and bat during contact.

$F\Delta t = m\Delta v$ $\quad \Delta v = v' - v = 52 - {}^{-}39 = +91\,m/s$

$F(3 \times 10^{-3}) = (.145)(91)$

$F = 4398\,N$

4. Calculate the force exerted on a rocket, given that the propelling gases are expelled at a rate of 1500 kg/s with a speed of 4.00×10^4 m/s (at the moment of takeoff). The force on the gas can be found from its change in momentum.

$F\Delta t = m\Delta v$ $\quad F = (1500\,\frac{kg}{s})(4 \times 10^4\,\frac{m}{s})$

$F = \frac{m\Delta v}{\Delta t}$ kg/s $\quad F = 6.0 \times 10^7\,N$

5. A golf ball of mass 0.045 kg is hit off the tee at a speed of 45 m/s. The golf club was in contact with the ball for 3.5×10^{-3} s. Find (*a*) the impulse imparted to the golf ball, and (*b*) the average force exerted on the ball by the golf club.

$F\Delta t = m\Delta v$

$m\Delta v = (.045)(+45) = 2.025 =$ kgm/s impulse and momentum

$F\Delta t = 2.025$

$F = 579\,N$

6. You are the design engineer in charge of the crashworthiness of new automobile models. Cars are tested by smashing them into fixed, massive barriers at 50 km/h (30 mph). A new model of mass 1500 kg takes 0.15 s from the time of impact until it is brought to rest. (*a*) Calculate the average force exerted on the car by the barrier. (*b*) Calculate the average deceleration of the car.

$\Delta v = 0 - 13.9 = -13.9\,\frac{m}{s}$

$F\Delta t = m\Delta v$

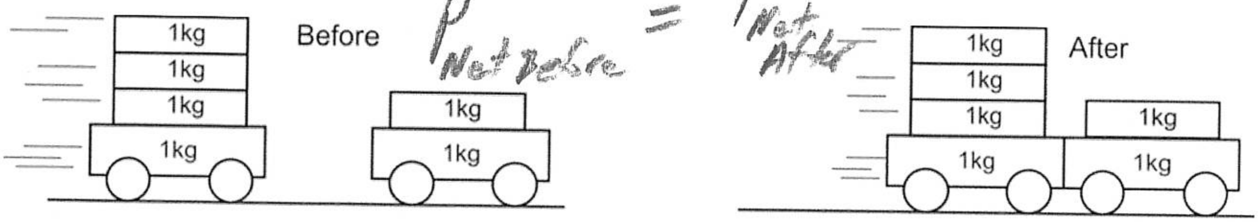
$F(.15) = (1500)(-13.9)$

$F = 139{,}000\,N$

$a = \frac{\Delta v}{\Delta t} = 93\,m/s^2$

7.

$P_{Net\,before} = P_{Net\,after}$



v=3m/s | v=0m/s | v=?

$p = 4kg(3\,m/s)$

$p = 12\,kg\,m/s$

$p = 0$

$P_{Net} = 12\,kg\,m/s$

$P_{Net} = mv$

$12\,kg\,m/s = 6kg(v) \Rightarrow V = 2\,m/s$

8.

v=0m/s $P_{Net} = 0$

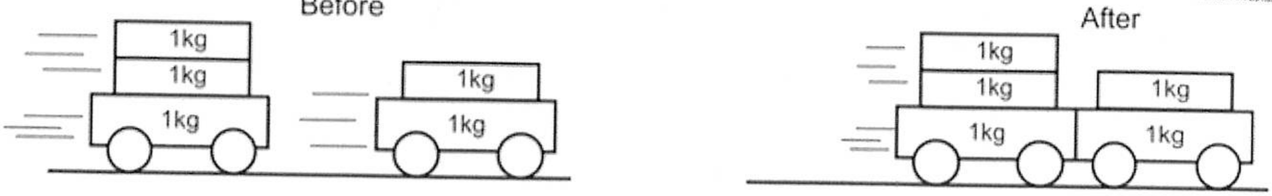
v=-2m/s

v=?

$P_{Net} = 0 = 2kg(-2\,m/s) + 1kg(v)$

$4\,kg\,m/s = 1kg(v) \Rightarrow v = 4\,m/s$

9.



v=5m/s | v=2m/s | v=?

$P_{Net} = 3kg(5\,m/s) + 2kg(2\,m/s) = 19\,kg\,m/s$

$19\,kg\,m/s = 5kg(v)$

$v = 3.8\,m/s$

6. You are the design engineer in charge of the crashworthiness of new automobile models. Cars are tested by smashing them into fixed, massive barriers at 50 km/h (30 mph). A new model of mass 1500 kg takes 0.15 s from the time of impact until it is brought to rest. (*a*) Calculate the average force exerted on the car by the barrier. (*b*) Calculate the average deceleration of the car.

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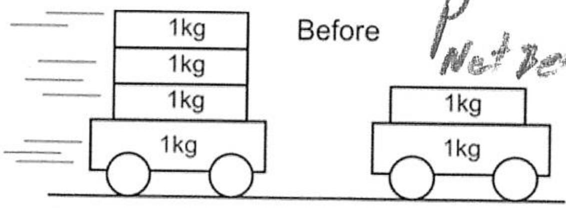
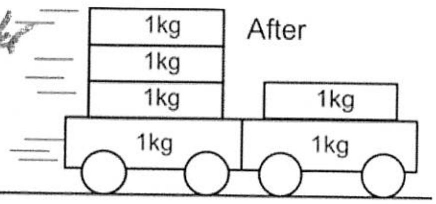
7.

Before



$P_{Net\ Before} = P_{Net\ After}$

After



v=3m/s   v=0m/s   v=?

$P = 4kg(3 m/s)$
$P = 12 kg\,m/s$

$P = 0$

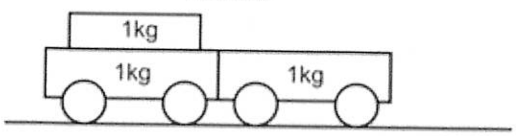
$P_{Net} = 12 kg\,m/s$

$P_{Net} = mv$

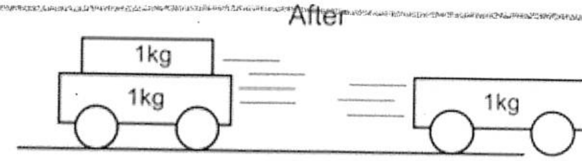
$12 kg\,m/s = 6kg(v) \Rightarrow V = 2 m/s$

8.

Before



After



v=0m/s $P_{Net} = 0$

v=-2m/s

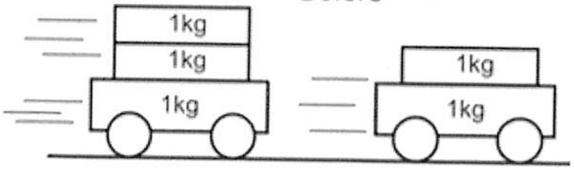
v=?

$P_{Net} = 0 = 2kg(-2m/s) + 1kg(v)$

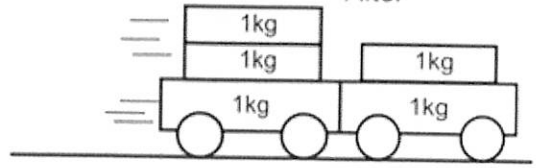
$4kg\,m/s = 1kg(v) \Rightarrow v = 4 m/s$

9.

Before



After



v=5m/s   v=2m/s   v=?

$P_{Net} = 3kg(5 m/s) + 2kg(2 m/s) = 19 kg\,m/s$

$19 kg\,m/s = 5kg(v)$

$v = 3.8 m/s$

10 1. A child in a boat throws a 6.40 kg package out horizontally with a speed of 10.0 m/s. Calculate the velocity of the boat immediately after, assuming that it was initially at rest. The mass of the child is 26.0 kg, and that of the boat is 45.0 kg. Ignore water resistance.

The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let "A represent the boat and child together, and let "B" represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)v = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_A = -\frac{m_B v'_B}{m_A} = -\frac{(6.40\text{ kg})(10.0\text{ m/s})}{(26.0\text{ kg} + 45.0\text{ kg})} = \boxed{-0.901\text{ m/s}}$$

11 2. A 12,600-kg railroad car travels alone on a level frictionless track with a constant speed of 18.0m/s. A 5350-kg load, initially at rest, is dropped onto the car. What will be the car's new speed?

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = (m_A + m_B)v' \rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(12{,}600\text{ kg})(18.0\text{ m/s}) + 0}{(12{,}600\text{ kg}) + (5350\text{ kg})} = \boxed{12.6\text{ m/s}}$$

12 3. A 3800 kg open railroad car coasts along level tracks with a constant speed of 8.60 m/s. Snow begins to fall vertically and fills the 3.50 kg/min. Ignoring friction with tracks, what is the speed of the car after 90 min?

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A = (m_A + m_B)v'_A$$

$$v'_A = \frac{m_A v_A}{m_A + m_B} = \frac{(3800\text{ kg})(8.60\text{ m/s})}{3800\text{ kg} + \left(\frac{3.50\text{ kg}}{\text{min}}\right)(90.0\text{ min})} = 7.94\text{ m/s} \approx \boxed{7.9\text{ m/s}}$$

d a velocity of 3m/s. Boat B has a mass of 15kg and a velocity of -1m/s. The
e away from one another. After the bounce, boat B has a velocity of 1.4m/s.

at A after the bounce?

$-0.6 m/s$

ced by boat A during the collision?

$= P_{Final} - P_{initial} = (-0.6 m/s)(10 kg) - (10 kg)(3 m/s) = -36 kg\,m/s$

by boat B?

$= -36 kg\,m/s$ $F = 360 N$

ced by boat B?

$5 m/s$

by boat A?

$-360 N$

restitution for this collision?

$e = \frac{V_B' - V_A'}{V_A - V_B} = \frac{1.4 m/s - (-0.6 m/s)}{(3 m/s) - (-1 m/s)} = \frac{2 m/s}{4 m/s} = 0.5$

nelastic?

hat is moving with a speed of 8.5 m/s collides head-on and elastically with
erward the incoming softball bounces backward with a speed of 3.7 m/s.
target ball after the collision, and (*b*) the mass of the target ball.

ing softball, and let B represent the ball initially at rest. The initial direction
ection. We have $v_A = 8.5\,\text{m/s}$, $v_B = 0$, and $v_A' = -3.7\,\text{m/s}$.
elationship between the velocities.

$$-(v_A' - v_B') \rightarrow v_B' = v_A - v_B + v_A' = 8.5\,\text{m/s} - 0 - 3.7\,\text{m/s} = \boxed{4.8\,\text{m/s}}$$

conservation to solve for the mass of the target ball.

$$v_B = m_A v_A' + m_B v_B' \rightarrow$$

$$\frac{(v_A - v_A')}{(v_B' - v_B)} = (0.220\text{ kg})\frac{(8.5\,\text{m/s} - -3.7\,\text{m/s})}{4.8\,\text{m/s}} = \boxed{0.56\text{ kg}}$$

15 5. Two bumper cars in an amusement park ride collide elastically as one approaches the other directly from the rear (Fig. 7–34). Car A has a mass of 450 kg and car B 550 kg, owing to differences in passenger mass. If car A approaches at 4.50 m/s and car B is moving at 3.70 m/s, calculate (*a*) their velocities after the collision, and (*b*) ~~the change in momentum of each.~~

5. Let the original direction of the cars be the positive direction. We have $v_A = 4.50\,\text{m/s}$ and $v_B = 3.70\,\text{m/s}$

(*a*) Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 0.80\,\text{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A + m_B v_B = m_A v'_A + m_B(0.80\,\text{m/s} + v'_A) \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B(v_B - 0.80\,\text{m/s})}{m_A + m_B} = \frac{(450\text{ kg})(4.50\,\text{m/s}) + (550\text{ kg})(2.90\,\text{m/s})}{1000\text{ kg}} = \boxed{3.62\,\text{m/s}}\ (b)$$

$$v'_B = 0.80\,\text{m/s} + v'_A = \boxed{4.42\,\text{m/s}}$$

Conceptual Questions

16 1. Why is momentum conserved for ALL collision, regardless of whether they are elastic or not?
Newton's 3rd Law says that each object feels the same force, but in opposite directions. By extension, they both feel equal and opposite impulses so the change on momentum is equal and opposite. One might say that one's loss is the other's gain.

because impact time is also equal

17 2. A Superball is dropped from a height h onto a hard steel plate (fixed to the Earth), from which it rebounds at very nearly its original speed. (a) Is the momentum of the ball conserved during any part of this process? (b) If we consider the ball and Earth as our system, during what parts of the process is momentum conserved?

(a) The momentum of the ball is not conserved during any part of the process, because there is an external force acting on the ball at all times – the force of gravity. And there is an upward force on the ball during the collision. So considering the ball as the system, there are always external forces on it, and so its momentum is not conserved.

(b) With this definition of the system, all of the forces are internal, and so the momentum of the Earth-ball system is conserved during the entire process.

18. PE of projectile + target at top of swing = $mgh = (1kg + 0.2kg)(9.8 m/s^2)(0.4m)$

$mgh = 4.70 J$

* Mechanical Energy is conserved during the swing

equal

$KE_{Proj+target}$ at bottom of swing = 4.70 J

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* Momentum is conserved during the collision

$KE = \frac{1}{2}mv^2$

$V_{proj+target}$ after collision $= \sqrt{\frac{2KE}{m}}$

$= \sqrt{\frac{2(4.70 J)}{1.2kg}} = 2.8 m/s$

$P_{projectile+target}$ after collision $= mv$

$= (1.2kg)(2.8 m/s)$

$= 3.36 kg\,m/s$

$P_{Net\ Before} = P_{Net\ After}$

$0.2kg(V) + 1kg(0) = 3.36 kg\,m/s \Rightarrow 16.8 m/s$

Projectile

target

Projectile in target