Physics 200 (Stapleton)
Circular Motion, Gravity, Kepler
Conceptual Practice, etc.

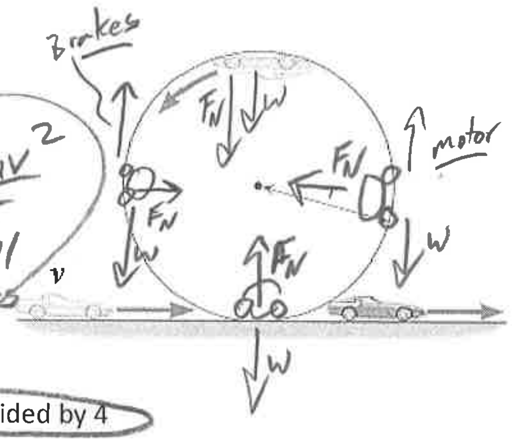
Name: Answers

Provided Formulas: $a_{centripetal} = v^2/r$ $F_{centripetal} = ma_{centripetal}$ $G = 6.67\text{x}10^{-11}\text{Nm}^2/\text{kg}^2$

$$F_{gravity} = G\left(\frac{M_1 m_1}{r^2}\right) \qquad \frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

1. A car is traveling at a constant speed on a loop-the-loop. Which of the following always represents the net force acting on the car while it is traveling in the circle?
   a. The centripetal force
   b. The weight of the car. ← True at top when $F_N = 0$
   c. The normal force ← True on sides
   d. The weight plus the normal force ← True at top
   e. The weight minus the centripetal force ← never true

$\sum F = \frac{mv^2}{r}$ at all times



2. If you move twice as far from the Earth's center, your weight...
   a. Doubles b. Quadruples c. Is divided by 2 d. Is divided by 4

3. Planet A is twice as far away from the Sun as Planet B. This means planet A's orbital period takes ____ as many Earth years.
   a. 4x b. 8x c. $\sqrt{8}x$ d. 16x

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3} \qquad \frac{T_A^2}{T_B^2} = \frac{(2)^3}{(1)^3} = \frac{8}{1} \qquad \sqrt{\frac{T_A^2}{T_B^2}} = \sqrt{8} = \frac{T_A}{T_B}$$

$$T_A = T_B\sqrt{8}$$

4. What is the difference between a satellite's altitude and its orbital radius?

Distance to Earth's surface

Distance to Earth's Center

5. A tetherball is rapidly orbiting a central pole. No outside force is being applied to the system. What is pulling the ball away from the pole?

Inertia

6. Starting with the Universal Law of Gravitation and centripetal force, derive a formula for the speed of a stable orbit around Earth. Show your steps.

$$\frac{mv^2}{r} = G\frac{m M_{Earth}}{r^2} \Rightarrow mv^2 = G\frac{m M_{Earth}}{r}$$

$$\Downarrow$$

$$v^2 = G\frac{M_{Earth}}{r}$$

$$v = \sqrt{G\frac{M_{Earth}}{r}}$$

7. Provide a formula for g on a planet of Mass M and radius r.

$$g = G\frac{M}{r^2}$$

8. A geostationary satellite orbits the Earth in a circular orbit while remaining over one fixed point on the equator. Can such a satellite orbit at a variety of altitudes or at just one altitude? Use one of Kepler's laws to explain your answer.

Just one altitude.

Kepler's 3rd Law → $\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_A^3}$

so ⇐ $\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$

For objects orbiting one planet, the ratio of period to radius is fixed. So, for a period of 1 day, there is only one possible radius.

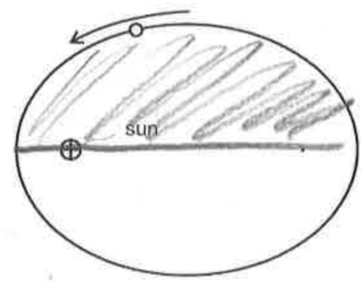
9. What does Kepler's 1st law tell us about planetary orbits?

Orbits are elliptical, with the sun at one focus of the ellipse

10. On the diagram to the right, label the areas where the planet is speeding up and slowing down.

speeding up ▨

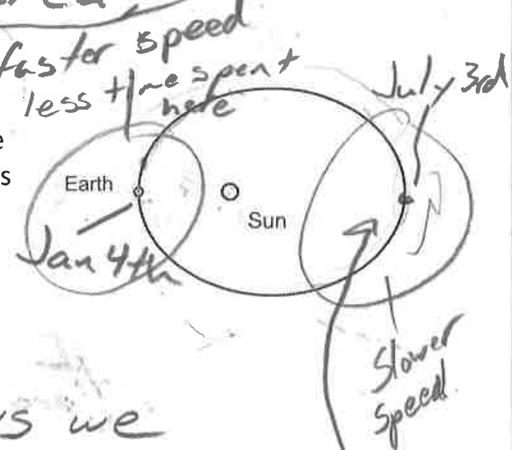
Slowing Down □



11 Explain why the planet is speeding up and slowing down in those areas of the diagram.

In the shaded areas, gravity is pulling in the same direction as the velocity. In the white area gravity opposes velocity

12. In 2017, the Earth was at Perihelion (closest to the sun) on January 4th. We will be at aphelion on July 3rd. In the Northern Hemisphere, are our winters longer than our summers, or are our Summers longer than our winters? Which of Kepler's Laws gives us the answer to this question? Explain your reasoning.

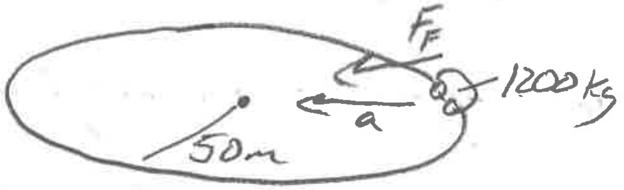


Summers are longer.

Kepler's 2nd Law (Equal Areas) says we move faster when we're closer to the sun, so we spend more time here, which is when we're experiencing Summer.

13. A 1,200kg car follows a semicircular curve at a constant speed. The static coefficient of friction of the car's tires against the track is μ_s=0.7. If the radius of the turn is 50m, and the surface of the track is horizontal, what is the maximum speed at which the car can round the corner without skidding?

$$\Sigma F = \frac{mv^2}{r} = F_{fr} \qquad F_{fr} = \mu mg = 0.7(1200kg)(9.8m/s^2) = 8232N$$



$$\Sigma F = \frac{1200kg(v^2)}{50m} = 8232N$$

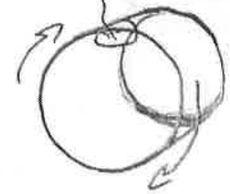
$$V = 18.5m/s$$

14. A side loading washing machine rotates around a horizontal axis. A 0.05kg undergarment at the top of its orbit during the spin cycle experiences an 4N normal force. The normal force is exerted by the inner wall of the washer drum. If the radius of the washing machine drum is 0.3m...

a. How many g's are being felt by the undergarment? 8.16g

b. What is the velocity of the undergarment in m/s? 5.19m/s

undergarment (0.05kg)



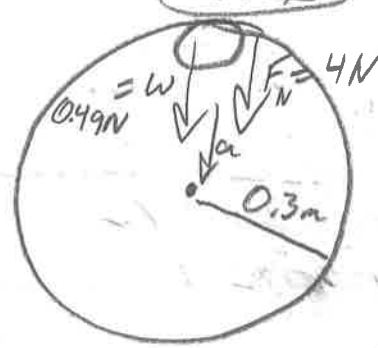
Garment "feels" the normal Force

$F_N = ma$

$4N = 0.05kg(a)$

$a = 80m/s^2$

$\frac{80m/s^2}{9.8m/s^2} = 8.16g$



$W = 0.05kg(9.8m/s^2)$

$W = 0.49N$

$$\Sigma F = \frac{-mv^2}{r} = -F_N - W$$

$$\frac{-0.05kg(v^2)}{0.3m} = -0.49 - 4N$$

$$v = 5.19m/s$$

16. What is the force of gravitational attraction between two 5kg [redacted] balls that have radii of 0.68m if they are touching one another?

These are huge bowling balls!

$$F_{grav} = G\frac{M_1 M_2}{r^2} = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\frac{(5kg)(5kg)}{(1.36m)^2}$$

$r = 2(0.68m) = 1.36m$

0.68m is the circumference of a bowling ball.

$$F_{grav} = 9.02\times10^{-10}N$$

16. Calculate g on Pluto, which has a mass of 1.3x10^{22}kg and a radius of 1,187km.

$$g = G\frac{M_{Pluto}}{r^2} = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\left(\frac{1.3\times10^{22}kg}{(1.187\times10^{6}m)^2}\right)$$

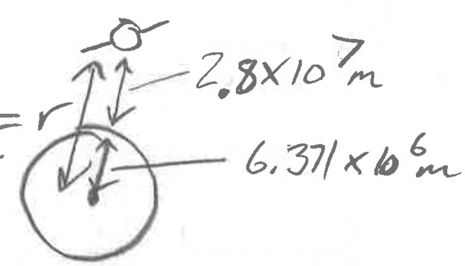
$$g = 6.15\times10^{-1}\,m/s^2 = 0.615\,m/s^2$$

17. The Earth's mass is 5.972x10^{24}kg and its average radius is 6.371x10^{6}m. Use these numbers to calculate the velocity of a satellite in a stable, circular orbit at a constant altitude of 28,000km above Earth's surface.

a. What is the satellite's orbital radius? $3.44\times10^{7}\,m$

b. What is the satellite's velocity? $3{,}402\,m/s$

$9.171\times10^{6}\,m = r$



$$v = \sqrt{G\frac{m_{Earth}}{r}}$$

$$v = \sqrt{6.67\times10^{-11}\frac{Nm^2}{kg^2}\left(\frac{5.972\times10^{24}kg}{3.44\times10^{7}m}\right)} = \sqrt{1.16\times10^{7}\frac{m^2}{s^2}}$$

$$v = 3{,}402\,m/s$$

18. a. What is the orbital period of the satellite in the previous question?
b. Use your previous answers to find the orbital radius of a geostationary satellite.

$$a.\ T = \frac{Circumf}{v} = \frac{2\pi r}{v} = \frac{2\pi(3.44\times10^{7}m)}{3{,}402\,m/s} = 63{,}533\ sec = 0.735\ days$$

b. A = Previous Satellite
B = Geostationary Satellite

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

$$\frac{(0.735\,day)^2}{(1\,day)^2} = \frac{(3.44\times10^{7}m)^3}{r_B^3}$$

$$r_B^3 = 7.54\times10^{22}\,m^3$$

$$r_B = 4.22\times10^{7}\,m$$