

Physics 200 (Stapleton) Unit 2 Handouts

Name: Key

Notes - 3.1. Kinematics in Two Dimensions: An Introduction

1. Give two examples of 2-dimensional motion.

A. Boat crossing a river (e.g. a "river problem")
at constant speed

B. The arc of a baseball (e.g. a "projectile motion" problem)

2. Given a right triangle of sides a and b and a hypotenuse of c , write the equation to find the length of c .

$$c = \sqrt{a^2 + b^2}$$

3. What is used to represent the magnitude and direction of a vector? an arrow

4. The length of the vector is directly proportional to the magnitude of the vector.

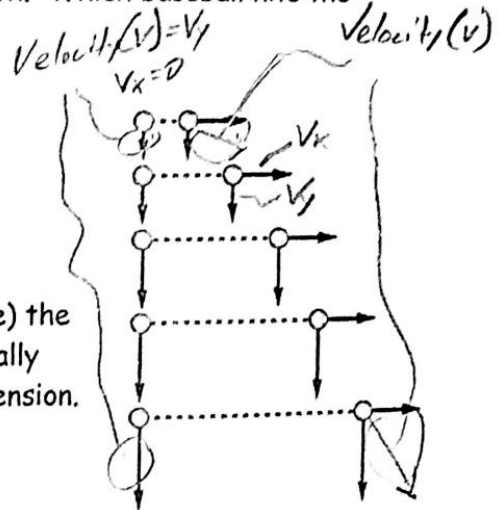
5. HUGE IDEA: The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

6. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. Which baseball hits the ground first? Neither (same time)

7. For the thrown (blue) ball in Figure 3.6 (on the right),
A. Is there acceleration in the y-direction?

B. Is there acceleration in the x-direction?

8. The key to analyzing such motion is to resolve (separate) the motion into separate X and Y vectors, which can be numerically added and subtracted with other vectors of the same dimension.



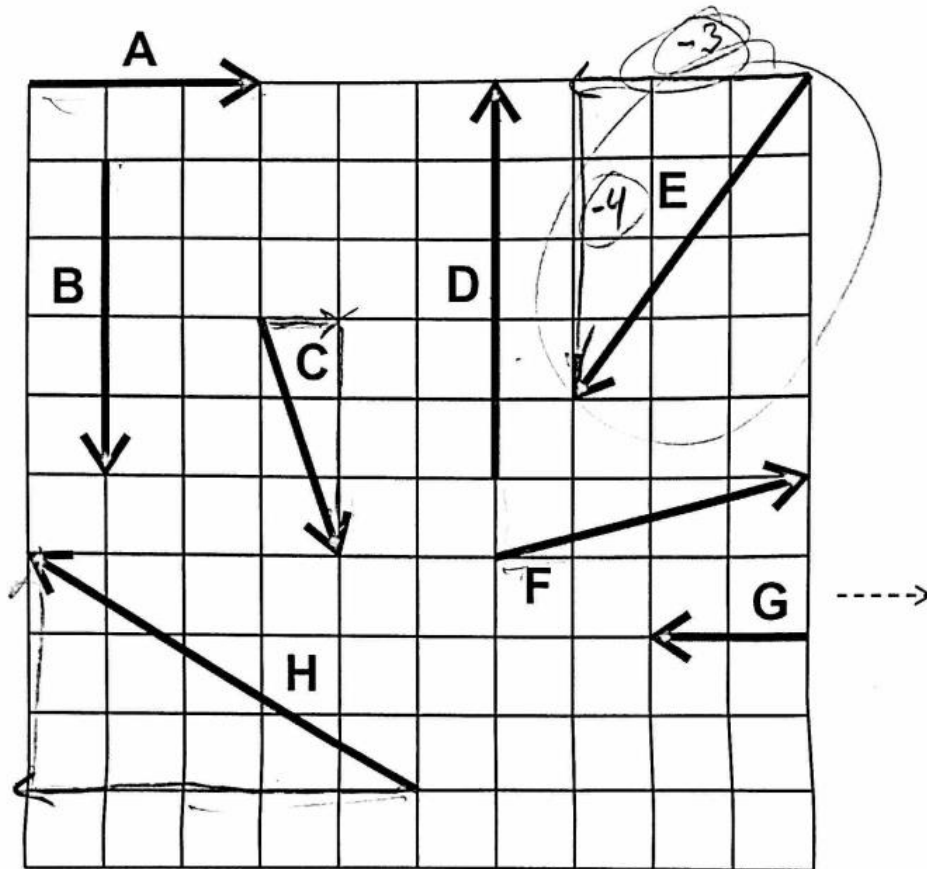
When a problem has zero acceleration in both the X and Y

dimensions, we call it a river problem. When a problem has zero acceleration in the X dimension, but an acceleration of -9.8m/s^2 in the Y dimension,

we call it a projectile problem.

②

Vector Addition Practice:



1. Find the resultant vector that is produced by adding vectors A and B.

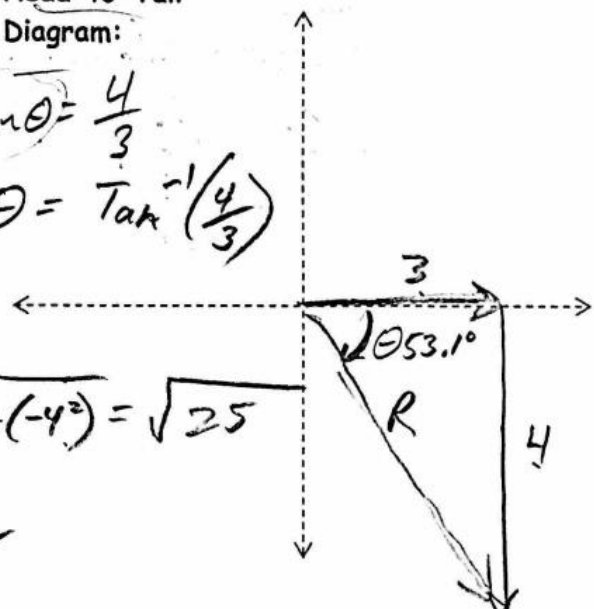
Vector	X comp.	Y comp.
A	3	0
B	0	-4
Totals	3	-4
Magnitude of Resultant	5	
Direction of Resultant	53.1° below positive x	

Head-to-Tail Diagram:

$$\tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\sqrt{3^2 + (-4)^2} = \sqrt{25}$$



3

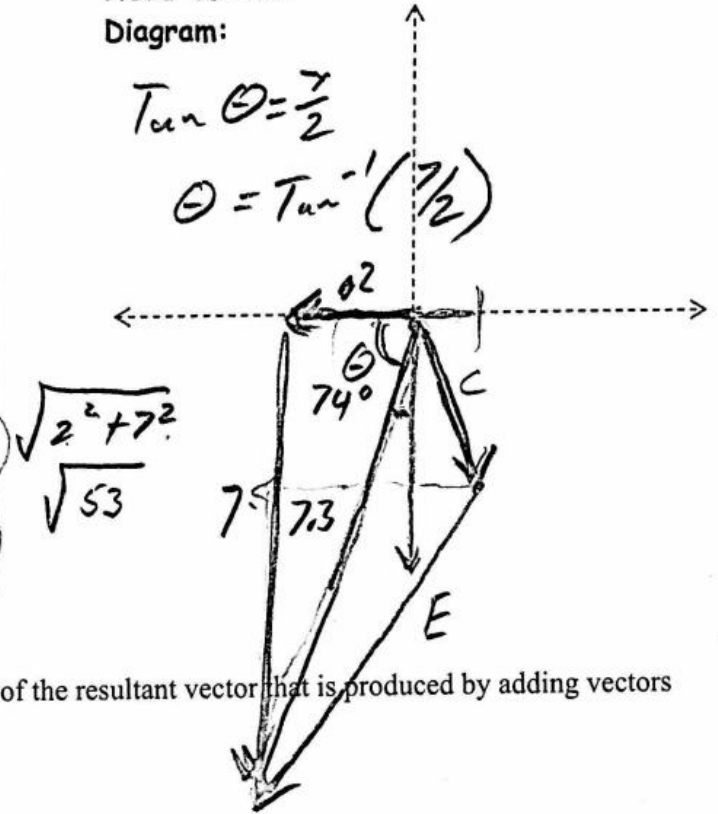
2. Add vectors E and C.

Vector	X comp.	Y comp.
E	-3	-4
C	+1	-3
Totals	-2	-7
Magnitude of Resultant	7.3	
Direction of Resultant	74° below -x	

Head-to-Tail Diagram:

$$\tan \theta = \frac{7}{2}$$

$$\theta = \tan^{-1}(7/2)$$



3. What is are the magnitude and direction of the resultant vector that is produced by adding vectors D, C, and A?

Find the resultant vectors from the additions of...

4. $E + H$

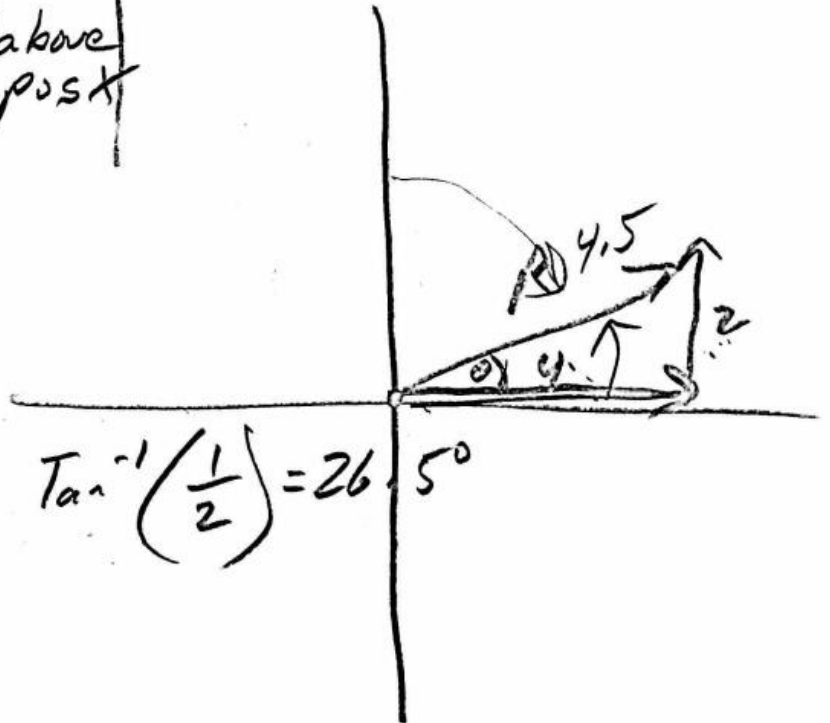
5. $C + F$

6. $E + H + G$

3

#3

	X	Y
D	0	5
C	1	-3
A	3	0
Totals	4	2
Res. Components		
Result (mag)	4.5	$\sqrt{16+4} = \sqrt{20}$
Result Direction	26.5° above post	

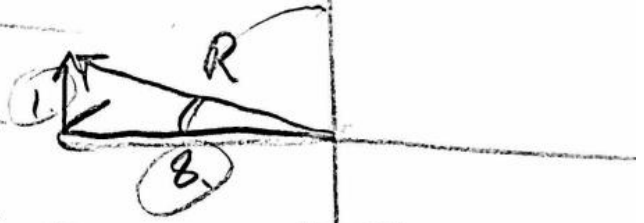


3

E+H
C+F
E+H6

#4

	X	Y
E	-3	-4
H	-5	+3
Res. components	-8	-1
R Mag	8.1	
R Dir	7.1° above neg X	



$$\sqrt{1 + 64} = \sqrt{65}$$

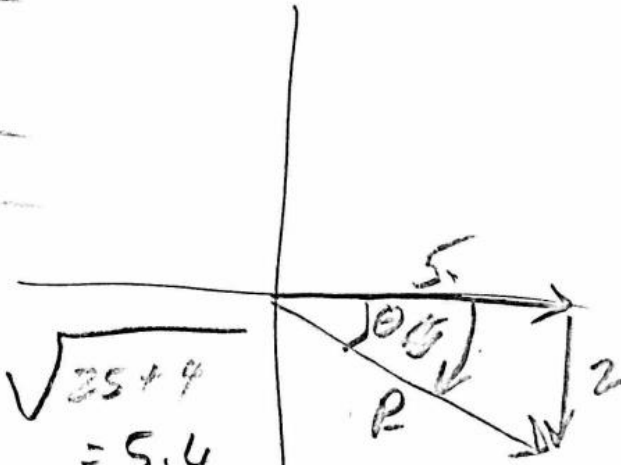
$$\tan^{-1}\left(\frac{1}{8}\right) = 7.1^\circ$$

#5

	X	Y
C	1	-3
F	4	1
R (comp)	5	-2
R Mag	5.4	
R Dir	21.8° below pos X	

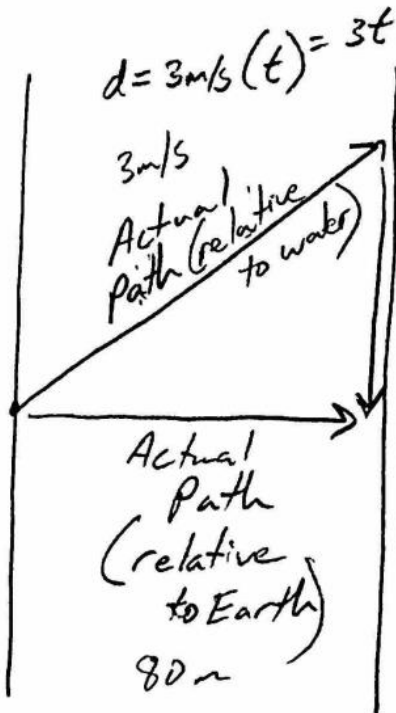
$$\sqrt{25 + 4} = 5.4$$

$$\theta = \tan^{-1} = \frac{2}{5} = 21.8^\circ$$



4

Jane



$$d = rt$$

Current $d = 2m/s(t) = 2t$
(2m/s)

~~$(2m/s)t$~~

$$(2t)^2 + 80^2 = (3t)^2$$

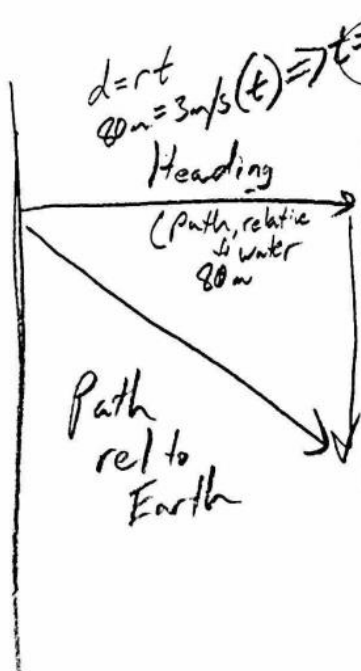
$$4t^2 + 6400 = 9t^2$$

$$6400 = 5t^2$$

$$t^2 = 1280$$

$t = 35.8s$

Bob



Current $d = rt$
(2m/s) $= 2m/s(26.7s) = 53.3m$

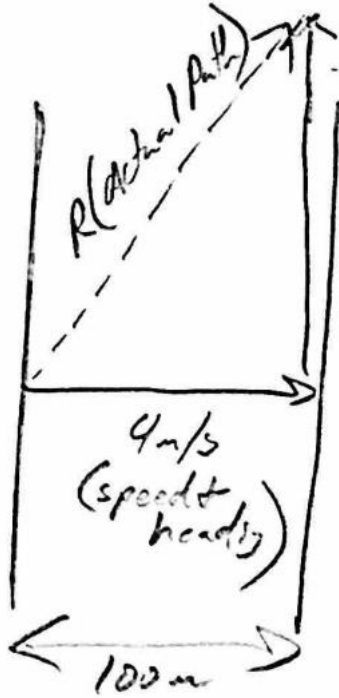
$$d = rt$$

$53.3m = 4m/s(t) \Rightarrow t = 13.3s$

$26.7s + 13.3s = 40s$

5

2.



7m/s (current)

$$a. R = \sqrt{(7\text{m/s})^2 + (4\text{m/s})^2}$$

$$R = 8.1\text{m/s}$$

$$b. \bar{v}_x = \frac{\Delta x}{\Delta t} \quad (\text{or } r = \frac{d}{t})$$

$$4\text{m/s} = \frac{100\text{m}}{\Delta t} \Rightarrow \Delta t = 25\text{s}$$

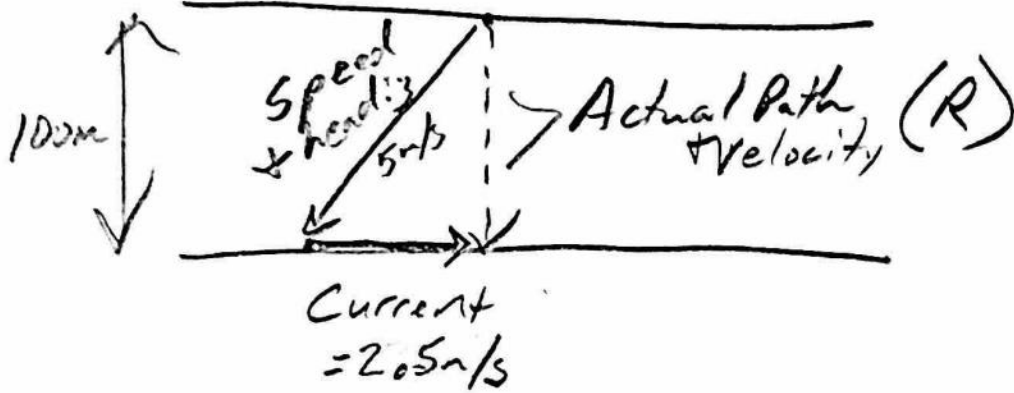
$$c. \bar{v}_y = \frac{\Delta y}{\Delta t} \quad (\text{or } r = \frac{d}{t})$$

$$-7\text{m/s} = \frac{\Delta y}{25\text{s}} \Rightarrow \Delta y = -175\text{m}$$

$$d = 175\text{m}$$

5

3.



$$R = \sqrt{(5 \text{ m/s})^2 - (2.5 \text{ m/s})^2} = 4.33 \text{ m/s}$$

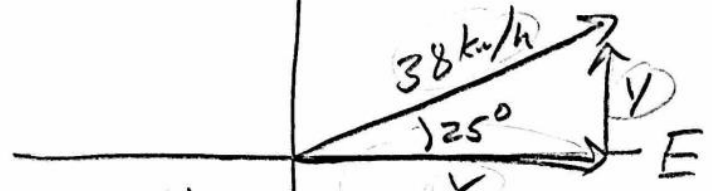
$$\bar{v}_y = \frac{\Delta y}{\Delta t}$$

$$4.33 \text{ m/s} = \frac{100 \text{ m}}{\Delta t}$$

$$\Delta t = 23.1 \text{ s}$$

5

#4 Wind Velocity (Component)



$$\sin 25^\circ = \frac{Y}{38 \text{ km/h}}$$

$$Y = 38 \text{ km/h} (\sin 25^\circ)$$

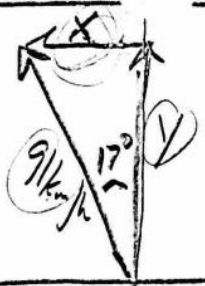
$$Y = 16 \text{ km/h}$$

$$\cos 25^\circ = \frac{X}{38 \text{ km/h}}$$

$$X = 38 \text{ km/h} (\cos 25^\circ)$$

$$X = 34.4 \text{ km/h}$$

Actual Copter Velocity (Resultant)



$$\cos 17^\circ = \frac{Y}{91}$$

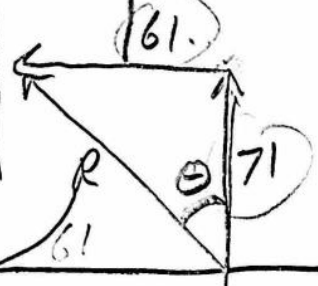
$$Y = 91 \text{ km/h} (\cos 17^\circ)$$

$$Y = 87 \text{ km/h}$$

$$X = 91 \text{ km/h} (\sin 17^\circ)$$

$$X = 26.6 \text{ km/h}$$

	X (km/h)	Y (km/h)
(C) Wind V	34.4	16
(C) Air Speed & Head	-61	71
(R) Actual Velocity	-26.6	87



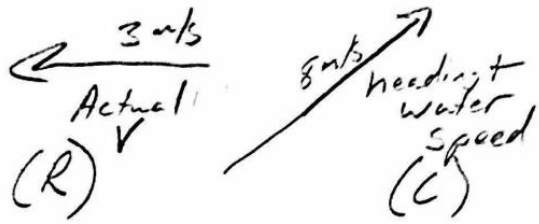
$$\text{Magnitude} = \sqrt{61^2 + 71^2} = 93.6 \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{61}{71}\right) = 40.7^\circ \text{ W of N}$$

or

$$49.3^\circ \text{ N of W}$$

5.

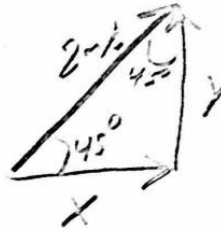


5

	X(m/s)	Y(m/s)
(C) Current	-8.66	-5.66
(C) Speed + Heading	5.66	5.66
(R) Actual V	-3	0

We know these

Subtract speed and heading from Actual V to find Current



$$\sin 45^\circ = \frac{y}{8 \text{ m/s}}$$

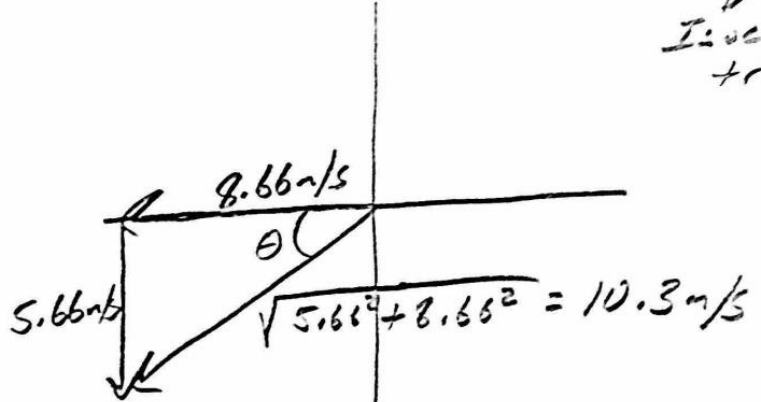
$$8 \text{ m/s} (\sin 45^\circ) = y$$

$$y = 5.66 \text{ m/s}$$

$$-3 - (5.66) = -8.66$$

$$0 - 5.66 = -5.66$$

$y = y = 5.66 \text{ m/s}$
Inverts triangle



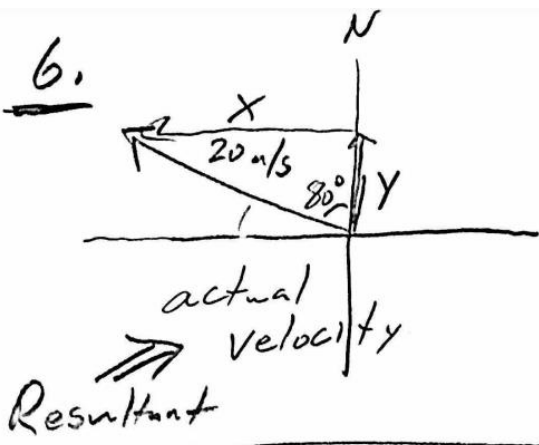
Current Magnitude = 10.3 m/s

$$\theta = \tan^{-1} \left(\frac{5.66 \text{ m/s}}{8.66 \text{ m/s}} \right) = 33.2^\circ \text{ below } -x$$

or
56.8° left of -y

5

6.

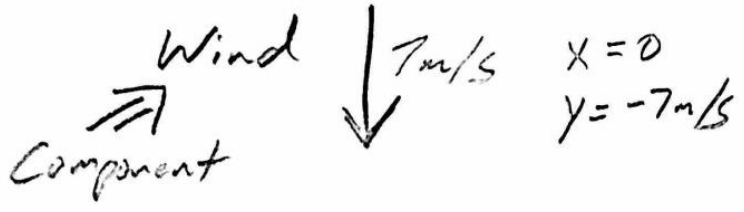


$$\sin 80^\circ = \frac{X}{20 \text{ m/s}}$$

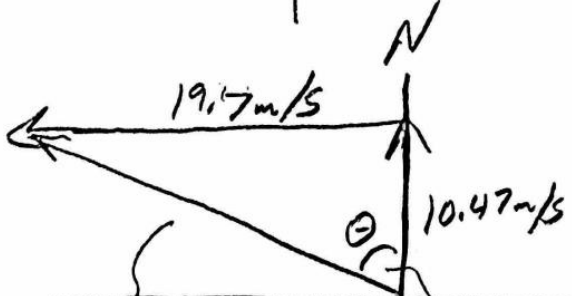
$$X = 20 \text{ m/s} \sin 80^\circ$$

$$X = 19.7 \text{ m/s}$$

$$Y = 20 \text{ m/s} (\cos 80^\circ) = 3.47$$



	X (m/s)	Y (m/s)
(C) Wind	0	-7
(C) Airspeed + heading	-19.7	10.47
(R) Actual Velocity	-19.7	+3.47



$$\sqrt{19.7^2 + 10.47^2}$$

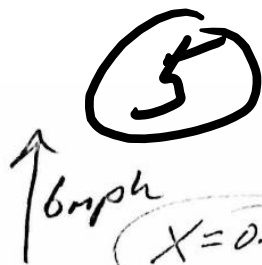
Magnitude = 22.3 m/s

$$\tan^{-1} \left(\frac{19.7}{10.47} \right) = 61.7^\circ \text{ W of N}$$

or

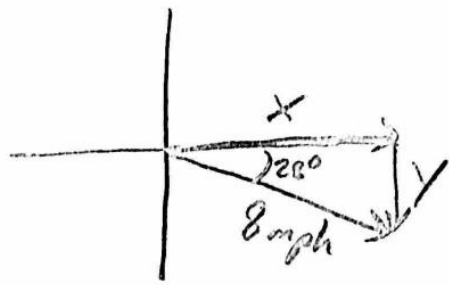
$$28.3^\circ \text{ N of W}$$

7.1 Cart Heading & Speed relative to Carrier Component



$X = 0 \text{ mph}$ $Y = 6 \text{ mph}$

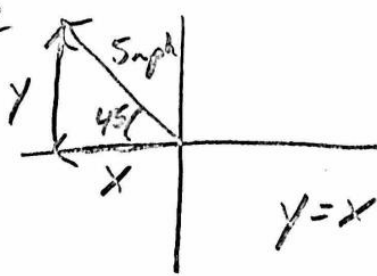
Carrier Heading & Water speed Component



$X = \cos 28^\circ (8 \text{ mph}) = 7.06 \text{ mph}$

$Y = \sin 28^\circ (8 \text{ mph}) = -3.76 \text{ mph}$

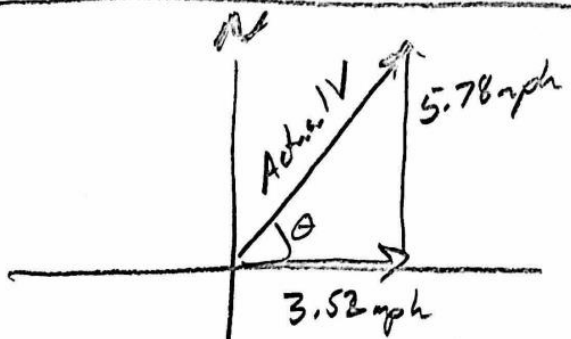
Ocean Current Component



$Y = X = \cos 45^\circ (5 \text{ mph}) = 3.54 \text{ mph}$

$X = -3.54 \text{ mph}$
 $Y = +3.54 \text{ mph}$

	X (mph)	Y (mph)
(C) Current	-3.54	3.54
(i) Carrier Heading & water spd	7.06	-3.76
(ii) Cart Heading & spd	0	6
(R) Actual Cart Velocity	3.52	5.78



Magnitude = $\sqrt{3.52^2 + 5.78^2} = 6.77 \text{ mph}$

$\theta = \tan^{-1} \left(\frac{5.78}{3.52} \right) = 58.7^\circ \text{ N of E}$
or $31.3^\circ \text{ E of N}$

6

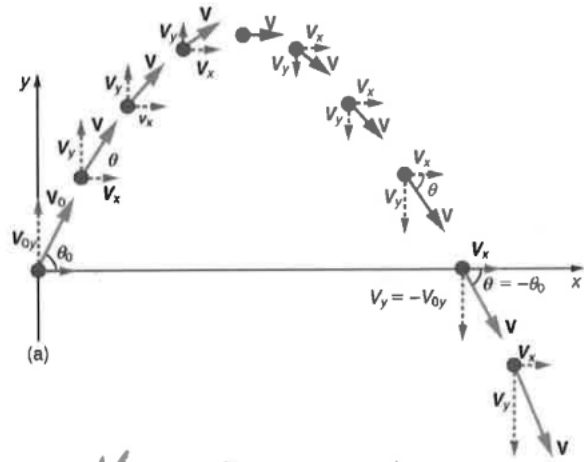
Name: Answers

Projectiles:

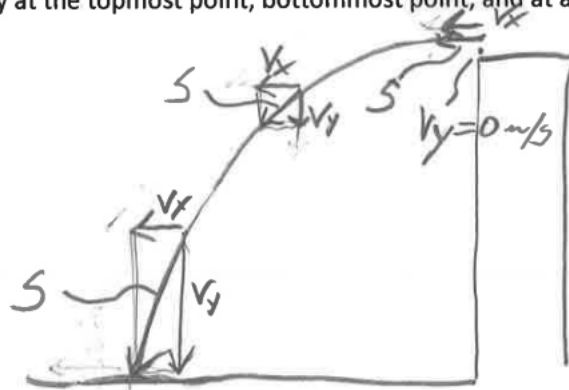
- The velocity of a launched projectile can be resolved into vertical (y) and horizontal (x) components. What happens to each of these components during the flight of the projectile? Why? Assume that there is no air resistance.

- V_y changes because gravity causes acceleration in y dimension.

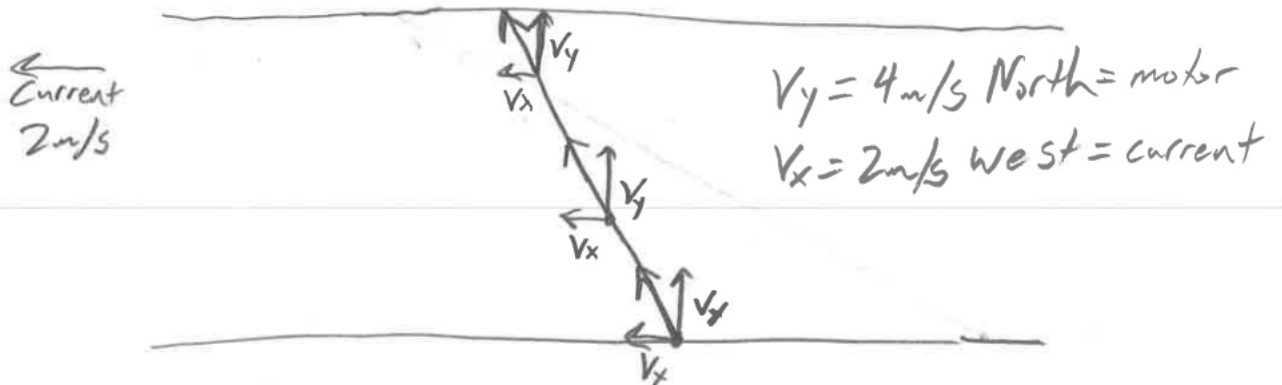
- V_x is constant because there is nothing causing horizontal acceleration



- A projectile is launched horizontally and to the left from the top of a tall building in the absence of air resistance. Sketch the path of the projectile as it falls to the ground. Use arrows to represent the object's speed, V_x , and V_y at the topmost point, bottommost point, and at a couple of other points in between.



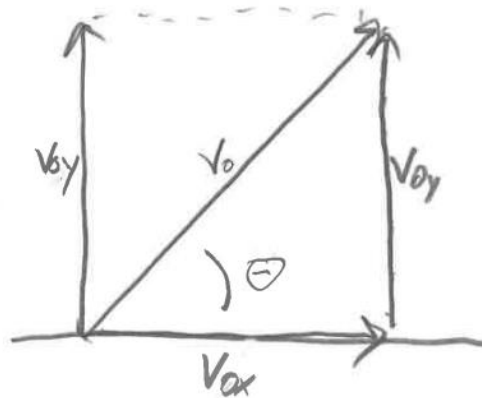
- Suppose a boat is launched directly northward across a river. The steering rudder is not adjusted once the trip is underway, and the boat's speed (relative to the water) is a steady 4m/s. The river's current has a westward velocity of 2m/s. What happens to the x and components of the boat's velocity as it crosses the river? Sketch a diagram showing the boat's path. For at least ~~two~~ two points, sketch vectors representing the boat's speed and velocity components (V_x and V_y).



7

A projectile is launched from ground level with an initial speed of V at an angle of θ above horizontal to the right. The projectile flies in the absence of air resistance until it returns to ground level.

4. Create a sketch showing the initial conditions in this problem. Show the initial velocity vector. Also resolve the initial velocity vector into X and Y components and sketch those components.



5. Use trig identities to provide the values of V_{0x} and V_{0y} .

$$V_{0x} = \cos \theta (V_0)$$

$$V_{0y} = \sin \theta (V_0)$$

6. Which component vector determines the time that the projectile remains in flight? Write a formula for time aloft.

V_{0y} determines time aloft

Time aloft = $\frac{2V_{0y}}{g}$ ← Twice the ascent time $\left(\frac{V_{0y}}{g}\right)$

7. Write a formula for the maximum height reached by the projectile.

$$\Delta y = V_{0y}(t) + \frac{1}{2}gt^2$$

$$\Delta y = V_{0y}\left(\frac{-V_{0y}}{g}\right) + \frac{1}{2}g\left(\frac{-V_{0y}}{g}\right)^2$$

$$\Delta y = \frac{V_{0y}^2}{g} - \frac{V_{0y}^2}{2g} = \frac{V_{0y}^2}{2g}$$

8. Write a formula for the distance traveled by the projectile. This is known as the range formula.

$$\Delta x = V_x (\text{time aloft})$$

$$= \cos \theta (V_0) \left(\frac{2V_{0y}}{g}\right)$$

$$= \cos \theta V_0 \left(\frac{2 \sin \theta V_0}{g}\right)$$

$$= \frac{V_0^2 2 \cos \theta \sin \theta}{g}$$

$$\frac{2 \sin \theta \cos \theta}{g} = \frac{\sin 2\theta}{g}$$

$g = 9.8 \frac{m}{s^2}$ Equations Provided On the Quiz

Resolving into x & y components:

Range formula:

$$\text{Range} = \frac{v_0^2 \sin 2\theta}{g}$$

Horizontal motion: $x = v_x t = v_0 (\cos \theta) t$

Vertical Motion:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 = y_0 + v_0 (\sin \theta) t - \frac{1}{2}gt^2$$

$$v_y = v_{y0} - gt = v_0 \sin \theta - gt$$

8

Projectile Practice Problems: Assume for all problems that there is no air resistance.

1. A car traveling at 60mph drives horizontally off of a cliff and falls to the ground 100m below.

a. Convert 60mph to m/s.

$$60 \text{ mph} \left(\frac{1 \text{ m/s}}{2.24 \text{ mph}} \right) = 26.8 \text{ m/s}$$

b. How long does it take the car to reach the ground?

$$V_{oy} = 0 \text{ m/s}$$
$$\Delta y = V_{oy}t + \frac{1}{2}(a)t^2$$
$$-100 \text{ m} = \frac{1}{2}(-9.8 \text{ m/s}^2)(t^2)$$
$$t = 4.52 \text{ s}$$



c. How far, horizontally, does the car fly through the air?

$$\Delta x = \bar{v}_x(t) = 26.8 \text{ m/s}(4.52 \text{ s}) = 121 \text{ m}$$

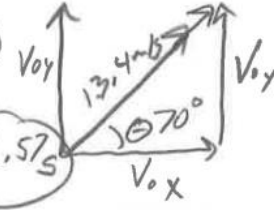
2. You throw a ball at a 70° angle with an initial speed of 30mph. The ball flies in an arc and lands on a shelf at the same height at which you released it.

a. Convert 30mph to m/s.

$$30 \text{ mph} \left(\frac{1 \text{ m/s}}{2.24 \text{ mph}} \right) = 13.4 \text{ m/s}$$

b. How long will the ball remain aloft before hitting the shelf?

$$V_{oy} = 13.4 \text{ m/s} (\sin \theta) = 13.4 \text{ m/s} (0.940)$$
$$V_{oy} = 12.6 \text{ m/s}$$
$$\text{Time aloft} = 2 \left(\frac{V_{oy}}{g} \right) = 2 \left(\frac{12.6 \text{ m/s}}{9.8 \text{ m/s}^2} \right) = 2.57 \text{ s}$$



c. What is the distance between the point of release and the point of impact on the shelf?

$$\Delta x = \bar{v}_x(\Delta t) = \cos(70^\circ)(13.4 \text{ m/s})(2.57 \text{ s})$$
$$= 0.34(13.4 \text{ m/s})(2.57 \text{ s})$$

$$\Delta x = 11.8 \text{ m}$$

d. What maximum height was reached by the ball?

$$\text{ascend time} = \text{time aloft} \div 2 = \frac{2.57 \text{ s}}{2} = 1.29 \text{ s}$$

$$\Delta y = V_{oy}t + \frac{1}{2}at^2 = 12.6 \text{ m/s}(1.29 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(1.29 \text{ s})^2$$
$$= 16.3 \text{ m} - 8.15 \text{ m}$$

$$\Delta y = 8.15 \text{ m}$$

9

3. You are trying to throw a ball through an open window that is 20m above the point at which you will release the ball and 5m in front of that release point. To minimize possible damage, you want the ball to enter the window at its apogee (max height). At what angle and with what initial speed should you release the ball?

1) Find V_{oy} $V_y^2 = V_{oy}^2 + 2a(\Delta y)$
 $0 \text{ m/s} = V_{oy}^2 + 2(-9.8 \text{ m/s}^2)(20 \text{ m})$
 $V_{oy} = 19.8 \text{ m/s}$

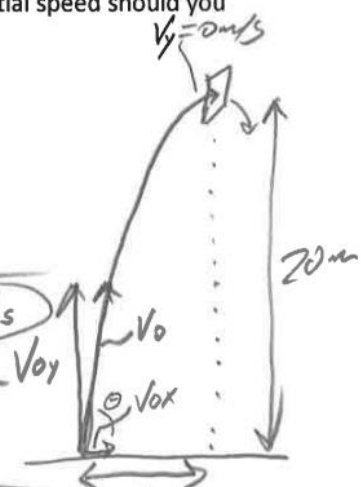
2) Find Ascent time $\Rightarrow \Delta t = \frac{V_{oy}}{g} = \frac{19.8 \text{ m/s}}{9.8 \text{ m/s}^2} = 2.02 \text{ s}$

3) Find V_{ox} $V_{ox} = V_x = \frac{\Delta x}{\Delta t} = \frac{5 \text{ m}}{2.02 \text{ s}} = 2.47 \text{ m/s}$

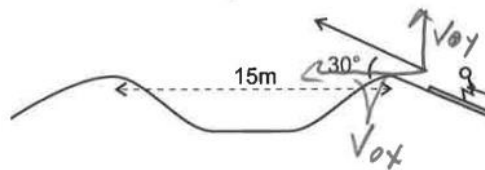
4) Find V_0 $V_0 = \sqrt{V_{x0}^2 + V_{y0}^2} = \sqrt{(2.47 \text{ m/s})^2 + (19.8 \text{ m/s})^2}$

~~$V_0 = 20.0 \text{ m/s}$~~ = initial speed

5) Find θ $\tan \theta = \frac{V_{oy}}{V_{ox}}$ $\theta = \tan^{-1}\left(\frac{V_{oy}}{V_{ox}}\right) = \tan^{-1}\left(\frac{19.8 \text{ m/s}}{2.47 \text{ m/s}}\right) = 83^\circ$
 above horizontal



4. A skier builds a jump and a landing area as shown on the diagram to the right. The takeoff point and the landing point are 15m apart and at equal elevations. The jump is inclined to horizontal at a 30 degree angle.



a. What speed does the skier need to attain in order to travel exactly 15 meters?

b. Given the initial speed from part a, what is the skier's maximum height, relative to the takeoff point?

c. Given the same initial speed, what is the skier's time aloft?

a) Using range formula ... $\text{range} = \frac{V_0^2 \sin 2\theta}{g}$ $15 \text{ m} = \frac{V_0^2 \sin(60^\circ)}{9.8 \text{ m/s}^2}$
 $\Rightarrow V_0^2 = \frac{15 \text{ m} (9.8 \text{ m/s}^2)}{\sin(60^\circ)} = 170 \text{ m}^2/\text{s}^2$ $V_0 = 13 \text{ m/s}$

b) $V_{oy} = \sin(30^\circ)(13 \text{ m/s}) = 6.5 \text{ m/s}$ $V_y^2 = V_{oy}^2 + 2a(\Delta y)$
 $0 \text{ m/s} = (6.5 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(\Delta y)$ $\Delta y = 2.16 \text{ m}$

c) Time aloft $= 2\left(\frac{V_{oy}}{g}\right) = 2\left(\frac{6.5 \text{ m/s}}{9.8 \text{ m/s}^2}\right) = 1.33 \text{ s}$

or $\Delta x = \Delta t (V_x) \Rightarrow 15 \text{ m} = \Delta t (\cos 30^\circ)(13 \text{ m/s}) \Rightarrow \Delta t = 1.33 \text{ s}$

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Name: Key

Chapter 3 Test 2015-2016

I. Multiple Choice: Select the one best answer for each question. Where g is used, assume it equals 10 m/s^2 and neglect air resistance for falling/moving objects.

1. For a symmetric projectile with an initial velocity of v_0 , what other angle gives the same range as 60° ?

- A) 5°
- B) 30°
- C) 45°
- D) 60°
- E) 75°

Angles are complementary
 $\theta_1 + \theta_2 = 90^\circ$

2. For a symmetric projectile with an initial velocity of v_0 , what angle gives the greatest range?

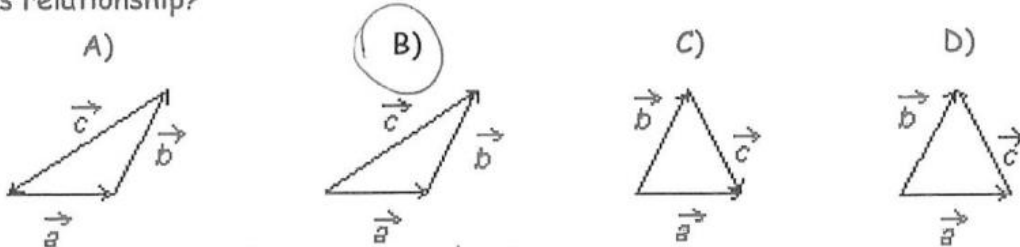
- A) 5°
- B) 30°
- C) 45°
- D) 60°
- E) 75°

3. A projectile is shot vertically upward with a given initial velocity. It reaches a maximum height of 50.0 m . If, on a second shot, the initial velocity is tripled (i.e. $3X$), then the projectile will reach a maximum height of:

- A) 75 m
- B) 100 m
- C) 150 m
- D) 200 m
- E) 450 m

$y_{\text{max}} = -\frac{v_{0y}^2}{2g} \Rightarrow y_{\text{max}} \sim v_{0y}^2$

4. The vectors \vec{A} , \vec{B} and \vec{C} are related by $\vec{C} = \vec{A} + \vec{B}$. Which diagram below illustrates this relationship?



Head-to-tail

5. A bird flies at a speed of 15 m/s with respect to the ground and the wind is blowing at a speed of 5 m/s with respect to the ground. [Note: The wind could be blowing with the bird, in the opposite direction of the bird or all other possible directions.] Which one of the speeds listed below is a possible net speed (i.e. vector sum) of the bird with respect to the ground?

- A) 3 m/s
- B) 5 m/s
- C) 9 m/s
- D) 18 m/s
- E) 25 m/s

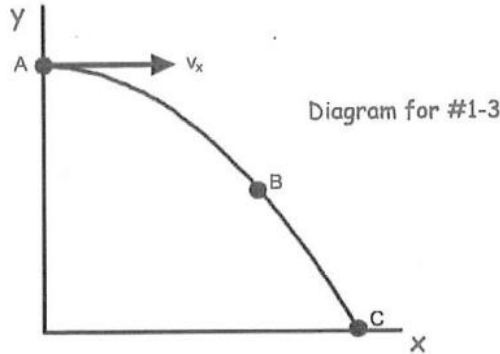
Min: $15 - 5 = 10 \frac{\text{m}}{\text{s}}$
 Max: $15 + 5 = 20 \frac{\text{m}}{\text{s}}$



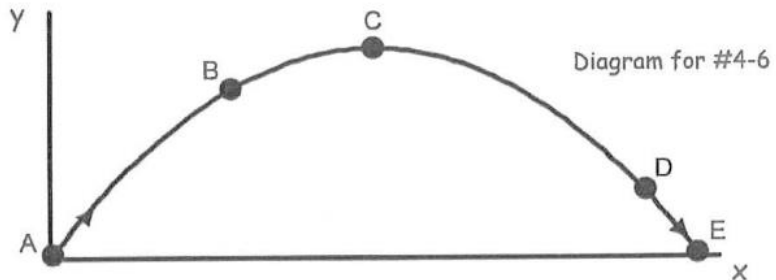
For #6-11, the answers will be $<$, $=$ or $>$, but you will mark A, B or C on your Scantron sheet. Assume no air friction for these projectiles and consider only the speed of the projectile (i.e. disregard the + and - signs).

A) $<$ B) $=$ C) $>$

- C 6. $v_B \geq v_{yA}$
- B 7. $a_a = a_B$
- A 8. $v_{yB} \leq v_{yC}$



- B 9. $v_{xC} = v_{xD}$
- B 10. $v_A = v_E$
- C 11. $v_{yD} \geq v_{yB}$



12. A vector has a component of 5 m in the $+x$ direction and a component of 12 m in the $+y$ direction. The magnitude of this vector is:

- (A) 13 m B) 15 m C) 17 m D) 60 m E) 169 m

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{5^2 + 12^2} = 13 \text{ m}$$

13. A vector in the xy plane has an x -component of 14.0 and a y -component of 9.4. The angle it makes with the positive x axis is:

- A) 26° (B) 34° C) 45° D) 59° E) 66°

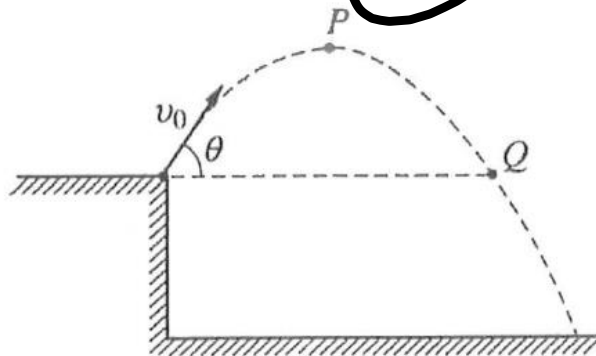
$$\theta = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{9.4}{14.0} = 34^\circ$$

14. Which of the following cannot be a vector quantity?

- A) velocity B) acceleration C) force (D) temperature

12

Diagram for Questions 15 & 16



A rock is thrown from the edge of a cliff with an initial velocity v_0 at an angle θ with the horizontal as shown above. Point P is the highest point in the rock's trajectory and point Q is level with the starting point. Assume air resistance is negligible.

15. Which of the following correctly describes the horizontal and vertical speeds and the acceleration of the point at Point P?

	<u>Horizontal Speed</u>	<u>Vertical Speed</u>	<u>Acceleration</u>
A)	$\rightarrow v_0 \cos \theta$	$\rightarrow 0$	$\rightarrow g$
B)	0	$\rightarrow 0$	$\rightarrow g$
C)	$\rightarrow v_0 \cos \theta$	$v_0 \sin \theta$	$\rightarrow g$
D)	0	$v_0 \cos \theta$	$\rightarrow g$
E)	$\rightarrow v_0 \cos \theta$	$\rightarrow 0$	0

16. Which of the following correctly describes the horizontal and vertical speeds and the acceleration of the point at Point Q?

	<u>Horizontal Speed</u>	<u>Vertical Speed</u>	<u>Acceleration</u>
A)	$v_0 \cos \theta$	0	$\rightarrow g$
B)	0	0	$\rightarrow g$
C)	$\rightarrow v_0 \cos \theta$	$\rightarrow v_0 \sin \theta$	$\rightarrow g$
D)	0	$v_0 \cos \theta$	$\rightarrow g$
E)	$v_0 \cos \theta$	0	0

13

17. A bullet shot horizontally from a gun. At the same instant, another bullet is simply dropped from the same height. Neglecting air resistance, the bullet shot from the gun

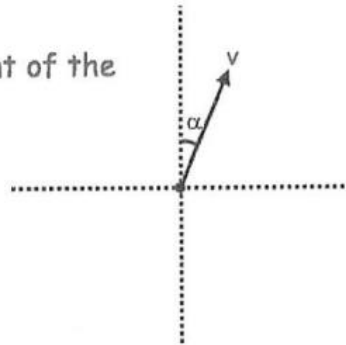
- A) strikes the ground much later than the dropped bullet.
- B) never strikes the ground.
- C) strikes the ground at the same time as the dropped bullet
- D) travels in a straight line.
- E) strikes the ground much sooner than the dropped bullet.

18. If θ is the angle with respect to the $+x$ -axis, the y -component of the vector A is given by

- A) $A \cos \theta$
- B) $\mu A \cos \theta$
- C) $A \sin \theta$
- D) $mg - A \sin \theta$
- E) $\tan^{-1} \theta$

19. Given the diagram to the right, what is the x -component of the vector v ?

- A) $V \sin \alpha$
- B) $V \cos \alpha$
- C) $V \tan \alpha$
- D) $V \sin^{-1} \alpha$
- E) $\sqrt{v_x^2 + v_y^2}$



14

Physics 200 Chapter 3 Test
2015-2016

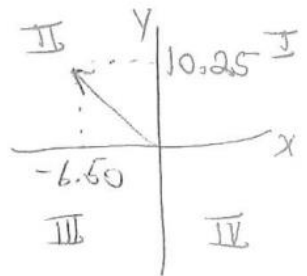
① $|\vec{F}| = 12.0\text{ N}$ $\theta = 290.0^\circ$

A. $x = F \cos \theta = 12.0\text{ N} \cos 290.0^\circ = \boxed{4.10\text{ N}}$

B. $y = F \sin \theta = 12.0\text{ N} \sin 290.0^\circ = \boxed{-11.3\text{ N}}$

② $r_x = -6.50\text{ m}$ $r_y = 10.25\text{ m}$

A. $r = \sqrt{r_x^2 + r_y^2} = \sqrt{(-6.50\text{ m})^2 + (10.25\text{ m})^2}$
 $= \boxed{12.1\text{ m}}$



B. $\theta = \tan^{-1} \frac{10.25\text{ m}}{-6.50\text{ m}} + 180^\circ = -57.62 + 180^\circ = \boxed{122^\circ}$
II Quadrant

③ $v_0 = 40.0 \frac{\text{m}}{\text{s}}$ $\theta_0 = 58.0^\circ$ $t = 4.60\text{ s}$ $x_0 = 0$ $y_0 = 0$

A. $x = x_0 + v_{0x}t = 0 + (40.0 \frac{\text{m}}{\text{s}} \cos 58.0^\circ)(4.60\text{ s})$
 $= \boxed{97.5\text{ m}}$

B. $y = y_0 + v_{0y}t + \frac{1}{2}gt^2 = 0 + (40.0 \frac{\text{m}}{\text{s}} \sin 58.0^\circ)(4.60\text{ s})$
 $+ \frac{1}{2}(-9.80 \frac{\text{m}}{\text{s}^2})(4.60\text{ s})^2$
 $= \boxed{52.4\text{ m}}$

14

④ $V_{0x} = 16.0 \frac{m}{s}$ $V_{0y} = 12.0 \frac{m}{s} \Rightarrow V_0 = 20.0 \frac{m}{s}$ $\theta = 36.87^\circ$

A. $y_{\max} = H = \frac{-V_{0y}^2}{2g} = \frac{-(12.0 \frac{m}{s})^2}{2(-9.80 \frac{m}{s^2})} = \boxed{7.35 m}$

B. $t = \frac{-2V_{0y}}{g} = \frac{-2(12.0 \frac{m}{s})}{-9.80 \frac{m}{s^2}} = \boxed{2.45 s}$

C. $R = \frac{V_0^2 \sin 2\theta_0}{g} = \frac{(16.0^2 + 12.0^2) \sin(2(\tan^{-1} \frac{12.0}{16.0}))}{9.80 \frac{m}{s^2}} = \boxed{39.2 m}$

D. $V_{up} = V_{down} \Rightarrow V = \sqrt{(16.0 \frac{m}{s})^2 + (12.0 \frac{m}{s})^2} = \boxed{20.0 \frac{m}{s}}$

⑤ $y - y_0 = 60.0 m$ $x - x_0 = 150.0 m$

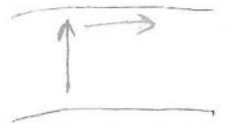
A. $y - y_0 = V_{0y}t + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(y - y_0)}{g}} = \sqrt{\frac{2(60.0 m)}{9.80 \frac{m}{s^2}}} = \boxed{3.50 s}$

B. $x - x_0 = V_{0x}t \Rightarrow V_{0x} = \frac{x - x_0}{t} = \frac{150.0 m}{3.50 s} = \boxed{42.9 \frac{m}{s}}$

⑥ $V_b = 10.0 \frac{m}{s}$ $V_c = 3.00 \frac{m}{s}$ $y - y_0 = 3.00 \times 10^3 m$

A. $y - y_0 = V_{0y}t \Rightarrow t = \frac{y - y_0}{V_{0y}} = \frac{3.00 \times 10^3 m}{10.0 \frac{m}{s}} = \boxed{30.0 s}$

B. $x - x_0 = V_{0x}t = (3.00 \frac{m}{s})(30.0 s) = \boxed{90.0 m}$



Chapter 3 Practice Test – Kinematics in 2D

Concepts (about 30-40% of points)

1. Suppose a projectile is launched at some non-vertical angle in the absence of air resistance. The projectile remains in freefall for several seconds before hitting the ground.

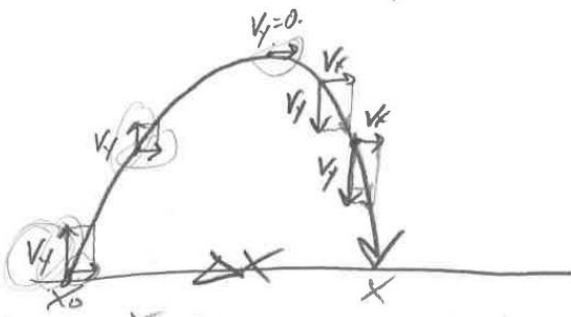
a. Choose any non-vertical launch angle and sketch the flight path of the projectile.

b. Describe what happens to the projectile's V_x over time and explain why. *Constant.*

No acceleration in X dimension

c. Describe what happens to the projectile's V_y over time and explain why.

Losing 9.8 m/s each second, because gravity is acting on the projectile

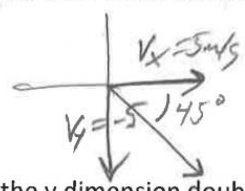


2. Can a projectile's Y velocity affect the projectile's X displacement? Why or why not?

Yes. A greater y velocity will extend the time aloft, giving more time for horizontal travel.

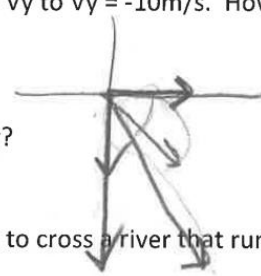
3. The velocity of an object moving in 2 dimensions has been resolved into two velocity components: $V_x=5\text{m/s}$ and $V_y = -5\text{m/s}$.

a. Describe the object's direction of travel.



b. Suppose the addition of a -5m/s air current in the y dimension doubles V_y to $V_y = -10\text{m/s}$. How does that change the object's speed?

Even faster.

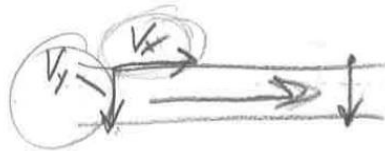


c. How does this this air current (from part b) affect the object's X velocity?

No effect

4. Which of the following determine(s) the amount of time it takes for a boat to cross a river that runs in the positive X direction?

- a. The river's current
- b. The boat's Y velocity Component
- c. The boat's X Velocity Component
- d. All of these
- e. None of these



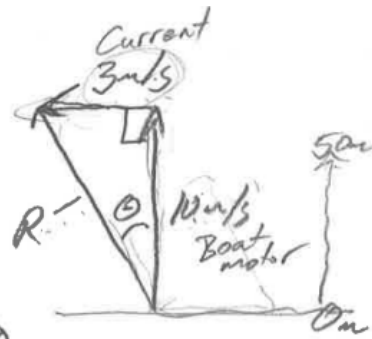
5. The figure on the right shows three vectors. Two of the vectors are component vectors that add up to the resultant vector. Which vector is the resultant vector? How can you tell?

If I arrange the other vectors head to tail, I get the middle as the resultant. R



16

Problems (about 60-70% of pts)



1. A motorboat with a water speed of 10 m/s and a Northward heading encounters a 3 m/s westward current.

a. What is the resultant velocity (not just speed) of the motorboat?

*see work on separate sheet

10.44 m/s 16.7° W of N

b. How long does it take the boat to travel to point that is 50m further north?

$$d = rt$$

$$50m = 10m/s(t) \quad t = 5s$$

c. In the time that the boat has traveled 50m northward, how far has it traveled to the west? (6 pts)

$$d = rt = 3m/s(5s) = 15m$$

2. A paddler wants to paddle in an eastward path across a river, ending up at a point directly across the river. The river is 100m wide, and it flows Northward with a current of 1.5m/s. In still water, the paddler's speed is 3m/s.

*work on separate sheet

a. What compass heading should the paddler follow? 30° S of W

b. Give the paddler's resultant velocity in terms of two component vectors, V_{North} and V_{East} .

$$V_{North} = 0m/s \quad V_{East} = 2.60m/s$$

3. A car drives horizontally off of a cliff. The cliff is 50m above the ocean below and the stunt driver wants the car to travel 60m, horizontally, before hitting the water. How fast should the car be traveling when it launches from the cliff?

*Work on separate sheet

18.75 m/s

4. A projectile is launched from ground level with a speed of 28m/s and a release angle of 72°. The projectile remains aloft until it returns to ground level.

*11

a. How long does the projectile remain aloft?

5.42s

b. What is the projectile's maximum height?

36m

c. How far, horizontally, does the projectile travel?

46.9m

d. What is the projectile's minimum speed during the flight (after release and before impact)?

8.65 m/s



16

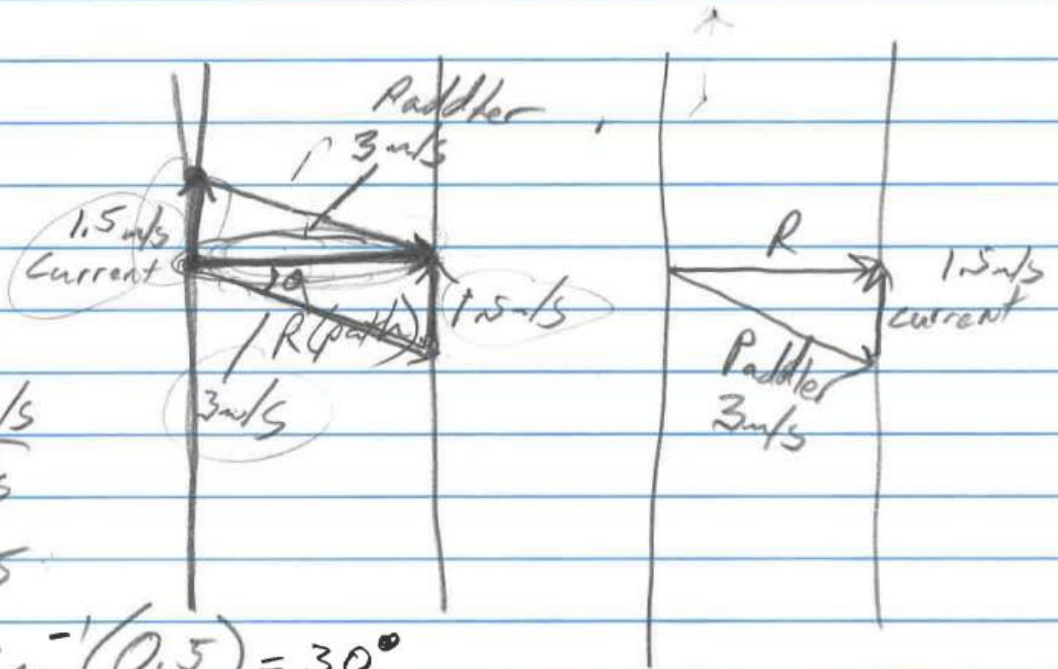
Problems

$$1. a) (10 \text{ m/s})^2 + (3 \text{ m/s})^2 = R^2$$

$$R = \sqrt{\frac{100 \text{ m}^2}{\text{s}^2} + \frac{9 \text{ m}^2}{\text{s}^2}} = 10.44 \text{ m/s}$$

$$\tan \theta = \frac{3}{10} \Rightarrow \tan^{-1}\left(\frac{3}{10}\right) = \theta = 16.7^\circ$$

2.



$$\sin \theta = \frac{1.5 \text{ m/s}}{3 \text{ m/s}}$$

$$\sin \theta = 0.5$$

$$\theta = \sin^{-1}(0.5) = 30^\circ$$

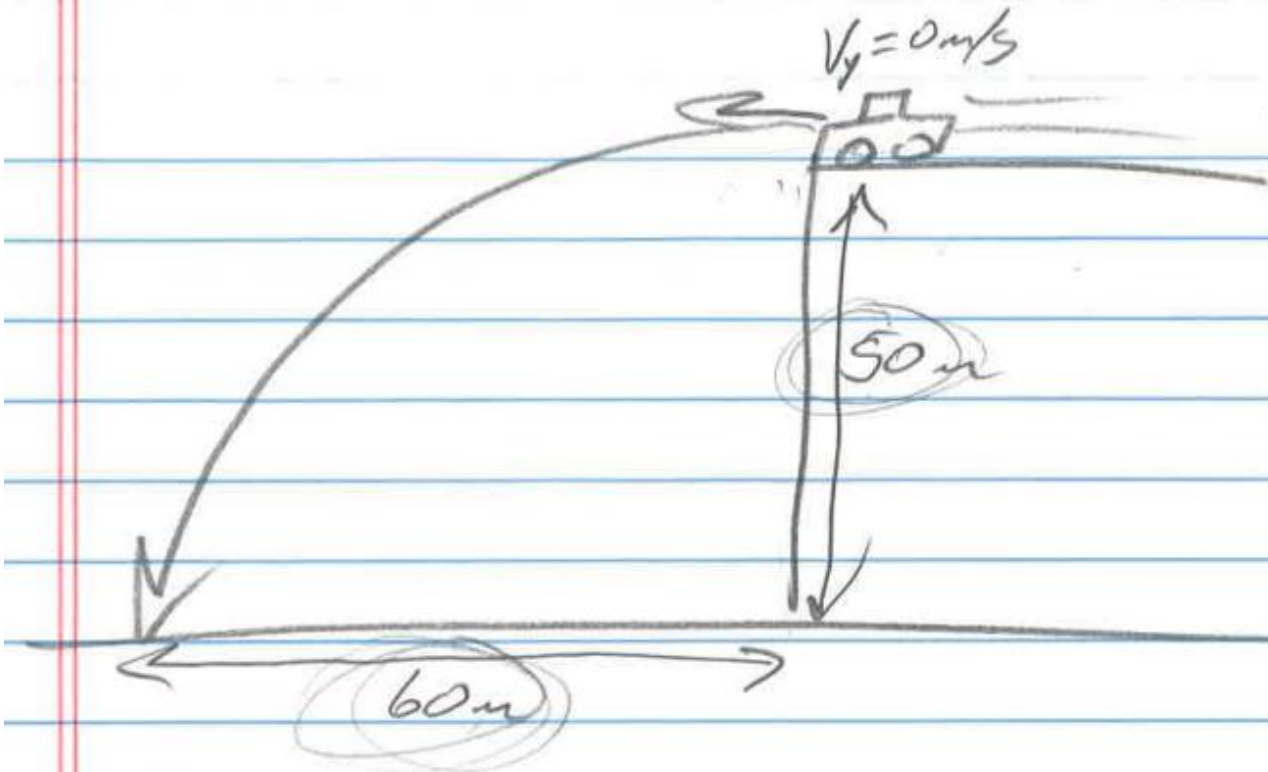
$$R^2 + (1.5 \text{ m/s})^2 = (3 \text{ m/s})^2$$

$$R^2 + \frac{2.25 \text{ m}^2}{\text{s}^2} = \frac{9 \text{ m}^2}{\text{s}^2}$$

$$R^2 = \frac{6.75 \text{ m}^2}{\text{s}^2} \quad R = 2.60 \text{ m/s}$$

3.

16



$$\Delta y = v_{oy}t + \frac{1}{2}at^2$$

$$-50\text{m} = \frac{1}{2}(-9.8\text{m/s}^2)(t^2)$$

$$t^2 = 10.2\text{s}^2$$

$$t = 3.2\text{s}$$

$$d = vt \quad \bar{v} = \frac{\Delta x}{\Delta t}$$

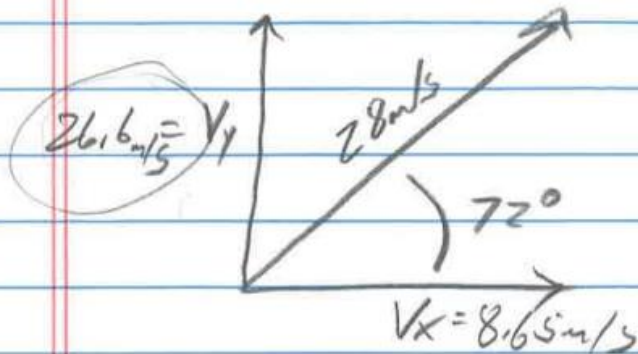
$$\Delta x = \bar{v} \Delta t$$

$$60\text{m} = \bar{v}_x(3.2\text{s})$$

$$\bar{v}_x = 18.75\text{m/s}$$

4.

16



$$V_y = (\sin 72^\circ) 28 \text{ m/s} = 26.6 \text{ m/s}$$

$$V_x = (\cos 72^\circ) 28 \text{ m/s} = 8.65 \text{ m/s}$$

a) $\frac{2V_y}{g} = 5.42 \text{ s}$

b) Ascent time = 2.71 s

$$\Delta y = V_0 t + \frac{1}{2} a t^2$$

$$= 26.6 \text{ m/s} (2.71 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2) (2.71 \text{ s})^2$$

$$\Delta y = 72.1 \text{ m} - 36.0$$

$$\Delta y = 36 \text{ m}$$

c) $\Delta x = \bar{V}_x \Delta t = (8.65 \text{ m/s}) (5.42 \text{ s}) = 46.9 \text{ m}$