Deriving $\Delta x = v_0 t + \frac{1}{2} a t^2$

$a = \frac{\Delta v}{\Delta t}$

$a = \frac{v - v_0}{\Delta t}$

Dropping the Δ for simplicity → $at = v - v_0$

$at + v_0 = v$

$\bar{v} = \frac{\Delta x}{\Delta t}$

$\bar{v} t \equiv \Delta x$ (dropped Δ for simplicity)

$\Delta x = \bar{v} t$

$\bar{v} = \frac{v_0 + v}{2}$

Substitute this for v

$\bar{v} = \frac{v_0 + (at + v_0)}{2}$

$\bar{v} = \frac{2v_0 + at}{2}$

↓ sub for $\bar{v}$

$\Delta x = \left(\frac{2v_0 + at}{2}\right) t$

$\Delta x = v_0 t + \frac{at^2}{2}$

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$