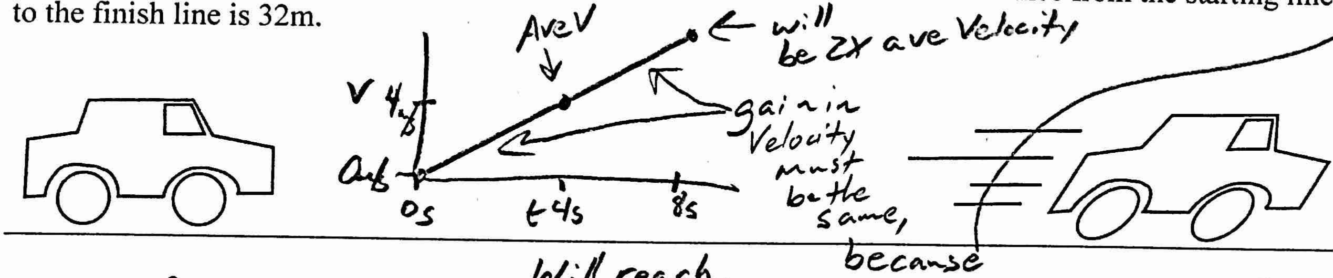
Physics 200
Notes: Intro to Kinematics Formulas

Name: Key

Deriving a formula for constant acceleration of an object starting from rest:

Suppose the car below moves with **constant acceleration** after starting from rest at the starting line. The car travels from the starting line to the finish line in a time of 8s. The distance from the starting line to the finish line is 32m.



Will reach $\bar{v}$ at t=4s

$\bar{v} = 4m/s$

$x_o = 0m$

$t_o = 0s$

$v_o = 0m/s$

$x = 32m$

$t = 8s$

$v = 8m/s$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{32m}{8s} = 4m/s$$

more generally,

$$\bar{v} = \frac{v + v_0}{2}$$

$$v = 2\bar{v} = 2(4m/s) = 8m/s$$

$$\Delta v = v - v_0 = 8m/s - 0m/s = 8m/s$$

$$\Delta t = t - t_0 = 8s$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8m/s}{8s} = 1m/s^2$$

Solving for a algebraically, using only symbols, we get…

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v}{\Delta t} = \frac{2\bar{v}}{\Delta t} = \frac{2\left(\frac{\Delta x}{\Delta t}\right)}{\Delta t} = \frac{2\Delta x}{\Delta t^2}$$

The formula we just derived for acceleration can be rearranged to give a version of the first displacement formula, on the right. What is the difference between the formula we derived and the formula on the right?

$a = \frac{2\Delta x}{t^2} \Rightarrow \frac{at^2}{2} = \Delta x$

The box on the right contains a more complete set of kinematics formulas for zero or constant acceleration. If you're curious about their derivations, many of the derivations can be found in the online textbook.

Those formulas will not always look like the ones on the right. Some symbols may be different, and some may be left out. For example:

$V_2 = V_0^{\,2} + 2a\Delta x$

$\Delta x = \bar{v} t$

$V = V_0 + at$

Formulas for zero or Constant Acceleration

$\bar{v} = \frac{\Delta x}{\Delta t}$ $\quad \bar{v} = \frac{v_{x0} + v_x}{2}$

$\bar{a} = \frac{\Delta v}{\Delta t}$

$\Delta x = v_{x0} t + \frac{1}{2} a t^2$

$\Delta x = \frac{1}{2}(v_{x0} + v_x) t$

$v_x = v_{x0} + at$

$v_x^2 = v_{x0}^2 + 2a(x - x_0)$

By convention, rightward and upward directions are positive, and leftward and downward directions are negative.

Practice With Motion Equations:

The G.U.E.S.S. method…

- Steps: Identify what is **G**iven. Identify the **U**nknown(s). Find an ***E**quation that incorporates the givens and the unknown. ****S**ubstitute givens into the equation. **S**olve.
- *Sometimes you will need more than one equation
- **It will sometimes save time and confusion if you solve for the unknown algebraically before substituting givens into the equation.

Example 1. What is the displacement of a car that starts from rest and accelerates at 6m/s^2 for 7 seconds?

$\Delta x = ?$ $\quad V_0$ $\quad a$ $\quad \Delta t$

$\Delta x = V_0 t + \frac{1}{2} a t^2 = \frac{1}{2}(6\,m/s^2)(7\,s)^2$

V_0: 0 m/s $\quad a$: 6 m/s^2 $\quad t$: 7 s

$= 147\,m$

Example 2. A nut is falling at a rate of -5m/s. If gravity accelerates the nut at a rate of -9.8m/s^2 for 6 additional seconds, what will be the velocity of the nut after those 6 seconds?

V_0 a Δt $V=?$

$V = V_0 + at$ ← 6s; V_0 = -5m/s; a = -9.8m/s^2

$V = -5\text{m/s} + (-9.8\text{m/s}^2)(6\text{s})$

$V = -63.8\text{m/s}$

1. A grouse takes off from the 8 yard line of a football field and travels to the 40 yard line over a time of 8 seconds. What is its average velocity?

x_0 x Δt $\bar{V}=?$

$$\bar{V} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{\Delta t} = \frac{40y - 8y}{8s} = \frac{32\text{yard}}{8s} = 4\,\frac{\text{yard}}{s}$$

2. A car accelerates from 30mph to 50mph over a time of 4 seconds. What is the car's average velocity during this time period? (Assume constant acceleration)

V_0 V t — Not necessary

$$\bar{V} = \frac{V_0 + V}{2} = \frac{30\text{mph} + 50\text{mph}}{2} = \frac{80\text{mph}}{2} = 40\text{mph}$$

3. You drop a rock off of a very high bridge. Starting from rest, the rock accelerates at a rate of -9.8m/s^2. You see a splash 8 seconds after you release the rock. What is the rock's velocity when it hits the water?

$V_0 = 0\text{m/s}$ a t V

$V = V_0 + at$

$V = 0\text{m/s} + 8s(-9.8\text{m/s}^2) = -78.4\text{m/s}$

4. Starting from rest, a car accelerates at 7m/s^2 for 10 seconds. What is the car's displacement during this time?

$V_0 = 0$ a t Δx

$\Delta x = V_0 t + \frac{1}{2}at^2$

$= 0t + \frac{1}{2}(7\text{m/s}^2)(10s)^2 = 350\text{m}$

5. A runner's velocity at 12:34:33 PM is 4m/s. At time 12:34:38 PM the runner's velocity is 2m/s. What is the runner's average acceleration over this time period?

t_0, V_0, t, V, Δt, $a = ?$

$$\bar{a} = \frac{\Delta V}{\Delta t} = \frac{V - V_0}{t - t_0} = \frac{-2\,m/s}{5s} = -0.4\,m/s^2$$

6. A child traveling at a rate of 15m/s along a zip line. After slowing down at a constant rate over a distance of 40m, the child comes to a stop.

V_0, Δx, $V = 0$

a. What is the child's average acceleration over this 40m?

$a = ?$

$$V^2 = V_0^2 + 2a\Delta x$$

$$0\,m/s^2 = (15\,m/s)^2 + 2a(40m)$$

$$0 = 225\,m^2/s^2 + 80m(a)$$

$$-225\,m^2/s^2 = 80m(a)$$

$$\frac{-225\,m^2/s^2}{80m} = a = -2.81\,m/s^2$$

b. How many seconds does it take for the child to travel this 40m distance?

$\Delta t = ?$

$$V = V_0 + at$$

$$0\,m/s = 15\,m/s + (-2.81\,m/s^2)t$$

$$t = 5.34s$$

7. (*requires multiple formulas) A driver sees a turtle in the road and hits the brakes. It takes him 3 seconds and a distance of 20m to slow to ~~a complete stop~~ 4m/s. What was the driver's rate of acceleration during this three second time period?

Δx, Δt, V, $a = ?$

$$a = \frac{\Delta V}{\Delta t} = \frac{V - V_0}{3s} = \frac{4\,m/s - V_0}{3s} \Rightarrow a = \frac{4\,m/s - 9.33\,m/s}{3s}$$

$$a = -1.78\,m/s^2$$

3 formulas

$$\bar{V} = \frac{\Delta x}{\Delta t} \text{ and } \bar{V} = \frac{V + V_0}{2}$$

so

$$\frac{\Delta x}{\Delta t} = \frac{V + V_0}{2}$$

$$\frac{20m}{3s} = \frac{4\,m/s + V_0}{2}$$

$$V_0 = 9.33\,m/s$$