

Momentum & Impulse: wrap-up and review

Coefficient of Restitution: tells how elastic a collision is; is a ratio of the separation speed of objects after a collision to their approach speed before the collision.

$$\text{Coefficient of Restitution} = \frac{\text{Separation Speed}}{\text{Closing speed}}$$

When $e=1$...

- objects separate as fast as they came together
- collision is **perfectly elastic**.
- No kinetic energy is lost.
- Example: A perfectly bouncy ball approaches the ground at 2m/s (closing speed) and then bounces back up with a speed of 2m/s (separation speed). $e = 2/2$

When $e=0$...

- objects do not separate
- the collision is **perfectly inelastic**.
- Kinetic energy is lost to friction.
- Example: a bullet approaches a ballistic pendulum at 500m/s (closing speed) and the bullet and pendulum then swing upward together (separation speed = 0; no separation). $e = 0/2$

Coefficient of Restitution Formula = $e = \frac{V_b' - V_a'}{V_a - V_b}$...where V_a = initial velocity of object A, V_b = initial velocity of object B, and V_a' and V_b' = their final velocities.

$$\text{When } e=1, V_b' - V_a' = V_a - V_b$$

Example Problem:

Cart A has a mass of 4kg and an initial velocity of -2m/s. Cart B has a mass of 3kg and an initial velocity of 0m/s. If the carts collide with perfect elasticity ($e=1$), what are the carts' velocities after the collision?

$$V_B' - V_A' = V_A - V_B \quad V_B' - V_A' = -2\text{m/s} - 0\text{m/s} = -2\text{m/s}$$

$$\underline{V_B' = V_A' - 2\text{m/s}}$$

Using Cons of P:

$$4\text{kg}(-2\text{m/s}) + 3\text{kg}(0\text{m/s}) = 4\text{kg}(V_A') + 3\text{kg}(V_B')$$

$$-8\text{kg m/s} = 4\text{kg } V_A' + 3\text{kg}(V_A' - 2\text{m/s})$$

$$-8\text{kg m/s} = 4\text{kg } V_A' + 3\text{kg } V_A' - 6\text{kg m/s}$$

$$-2\text{kg m/s} = 7\text{kg } V_A'$$

$$\underline{V_A' = -0.29\text{m/s}}$$

$$V_B' = -0.29\text{m/s} - 2\text{m/s} = \underline{-2.29\text{m/s}}$$

12. Boat A has a mass of 10kg and a velocity of 3m/s. Boat B has a mass of 15kg and a velocity of -1m/s. The two boats collide and bounce away from one another. After the bounce, boat B has a velocity of 1.4m/s.

a. What is the velocity of boat A after the bounce?

$$-0.6 \text{ m/s}$$

b. What impulse is experienced by boat A during the collision?

$$F \Delta t = \Delta p = p_{\text{final}} - p_{\text{initial}} = (-0.6 \text{ m/s})(10 \text{ kg}) - (10 \text{ kg})(3 \text{ m/s}) = -36 \text{ kg m/s}$$

c. What impact force is felt by boat B?

$$F(0.1 \text{ s}) = -36 \text{ kg m/s} \quad F = 360 \text{ N}$$

d. What impulse is experienced by boat B?

$$36 \text{ kg m/s}$$

e. What impact force is felt by boat A?

$$F = -360 \text{ N}$$

f. What is the coefficient of restitution for this collision?

$$e = \frac{v_B' - v_A'}{v_A - v_B} = \frac{1.4 \text{ m/s} - (-0.6 \text{ m/s})}{(3 \text{ m/s}) - (-1 \text{ m/s})} = \frac{2 \text{ m/s}}{4 \text{ m/s}} = 0.5$$

g. Is the collision elastic or inelastic?

$$e < 1$$

13. A 0.2kg projectile is fired at a 1kg ballistic pendulum. After the projectile embeds in the target, the pendulum and projectile swing upward a height of 0.4m. What was the initial velocity of the projectile (just before it hit the pendulum)?

$$16.8 \text{ m/s}$$

14. A catcher catches 0.145kg baseball traveling at 35m/s. If the catcher applies a stopping force of 500N to the ball, over what length of time does the catcher feel the ball's impact?

$$F \Delta t = \Delta p = m \Delta v$$

$$-500 \text{ N} (t) = 0.145 \text{ kg} (-35 \text{ m/s})$$

$$t = 0.01 \text{ s}$$